Digital Goods Reselling: Implications on Cannibalization and Price Discrimination

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The resale of used products presents the challenge of cannibalization, particularly pronounced in digital goods markets where perfect substitutes are easily replicable. In this paper, we assert that, rather than a threat, resale can serve as an effective pricing tool for managing heterogeneous demand. We consider a seller of digital goods/services who offers a contract to a heterogeneous group of customers at a fixed price for a specified amount of usage allowance. Rather than imposing restrictive sharing barriers, the seller allows subscribers to share their allowances with others in a secondary market. Our analysis reveals that the seller's optimal strategy involves facilitating resale by eliminating transaction costs. The sharing contract effectively achieves the same outcome as a two-part tariff, wherein subscribers pay an entry fee along with a marginal usage rate. Both approaches generate equivalent revenue and market coverage, and result in idential demand and individual surplus for customers of the same type. Consequently, the sharing contract acts as a mechanism for price discrimination. Our finding provides a new perspective on peer-to-peer resales and also challenges the conventional belief that successful price discrimination hinges on preventing resale.

1. Introduction

Conventional wisdom suggests that the resale of used physical goods, especially durable ones, provides close substitutes for new goods and thus could cannibalize sellers' profits in the primary market. Empirical studies across various industries (e.g., Ghose et al. 2006 for used books, Chen et al. 2013 for automobiles and Shiller 2013 for video games) also support this proposition. Firms are often forced to take substantial efforts in curtailing such cannibalization. Common strategies include planned obsolescence (Bulow 1982)—deliberate reduction of product durability, retail price markup of new goods (GAO 2005), and frequent product updates (Yin et al. 2010).

In digital goods markets, cannibalization may be exacerbated for two main reasons: (1) sharing a copy of the digital good does not prevent the sharer from enjoying it, and (2) shared or used digital goods are perfect substitutes for new ones. To address the first issue, digital rights management (DRM) technologies have been developed to prevent unauthorized access to the product or service. For instance, e-books downloaded to a Kindle account cannot be transferred to another device. However, the second problem poses a greater challenge because a shared digital good functions identically to the original copy. In this scenario, a primary market seller faces significant revenue loss as the seller of the used product can offer a perfect substitute at a lower price.

However, digital goods also exhibit substantial distinctions from physical goods, and these unique characteristics may offer potential solutions to cannibalization. On one hand, the resale of many digital goods, such as downloaded video games and mobile data, can be effortlessly traced via seller re-licensing and re-authorization, whereas obtaining such records for physical products once they are sold is nearly impossible. This traceability of sold digital products makes it technologically feasible for sellers to participate in and monetize the secondary market. On the other hand, sellers intending to participate in the secondary market of digital goods have more tools at their disposal. One such tool is the control over usage allowance, which is often a critical component of digital products, such as mobile data for telecommunication services and storage space for cloud services.The usage allowance provides sellers with a new avenue to navigate not only the primary market but also the secondary market.

An increasing number of sellers have recognized the unique characteristics of digital goods and begun allowing peer-to-peer resales. China Mobile Hong Kong (CMHK) launched an experimental pricing scheme called "sharing pricing" in 2013. Under this scheme, subscribers of CMHK's monthly data plan, which offers a fixed allowance at a specified price, can resell unused allowances to other subscribers via peer-to-peer trading on CMHK's platform. As of mid-2018, it was estimated the platform facilitates daily transactions in the magnitude of 10,000 GBs at an average price of HK$15 (approximately US$2) per GB (Huang et al. 2021).

This innovative pricing model diverges from the conventional approach to price discrimination (e.g., two- and three-part tariffs) adopted by most carriers. It also appears to challenge the conventional belief that the success of price discrimination hinges critically on a firm's ability to prevent a resale market—a perceived necessary condition for the effectiveness of nonlinear pricing (e.g., Oi 1971, Wilson 1993). (The quantity discount feature of nonlinear pricing can create arbitrage opportunities, where a customer buying a large order can profit by splitting it into several smaller lots and reselling them to others.) It seems paradoxical that a firm capable of implementing price discrimination with tightly restricted resale would instead embrace resale, potentially jeopardizing its success.

Similar instances of digital goods resale are evident in cloud storage services. Rather than adhering to industry norms of offering bucket contracts that entail a fixed monthly fee for a specific storage amount[1], Livedrive, a subsidiary company of the Nasdaq-listed digital media company Ziff Davis specializing in online cloud backup and storage service, provides a subscription plan for 5TB of cloud storage at £39.95 per month[2] with a resale feature that enables subscribers to resell storage allowances and share their total available space with others.

The above examples clearly illustrate the divided opinions among practitioners regarding the resale of digital goods. Considering the advantages and disadvantages of digital goods, it remains unclear from the literature to what extent their unique characteristics would support or hinder resales.

In this paper, we use a game-theoretic model to quantify a seller's strategy in a secondary market when the unique characteristics of digital goods can be exploited. Specifically, we consider a digital goods seller who offers heterogeneous customers a contract of a digital good/service (e.g., e-books, mobile data, or storage space) by charging a fixed price for a specific amount of usage allowances. The novel feature of this contract is that subscribers are allowed to share their allowances to others in a peer-to-peer resale market at a transaction cost that is determined by the seller. We thus refer to this contract as a sharing contract.

We characterize the optimal sharing contract and evaluate its performance. Contrary to existing literature, we find that the digital-goods seller's optimal strategy is to charge zero transaction cost. In other words, the seller should consciously promote resale and should not make any profit from resale transactions. We show that the optimal sharing contract performs the same as the optimal two-part tariff, in which subscribers pay an entry fee for access to the service and a marginal rate of usage. Not only do the two contracts yield identical revenue, they also result in the same market coverage and the same demand and individual surplus for customers of the same type. Therefore, the sharing contract establishes price discrimination and is essentially a form of nonlinear pricing.

Our findings demonstrate how the distinctive features of digital goods can be utilized to address challenges arising from digitization. In our model, the digital goods seller has the technical capability to monitor all secondary-market allowance exchanges, ensuring that no resale revenue goes unnoticed. However, despite this traceability, the seller does not tax on each resale at all. This outcome suggests that traceability of digital goods is not crucial for managing resales, as the seller does not profit from it at optimality.

[1] For example, Dropbox, a file hosting service provider, offers 2TB of cloud storage for $9.99 a month.

[2] https://www2.livedrive.com/ForResellers

We argue that the usage allowance effectively prevents profit loss from cannibalization: the seller utilizes the allowance in conjunction with the price to regain revenue upon initial sale of the sharing contract. Specifically, the seller determines the optimal allowance to ensure that the total supply to all subscribers matches the total usage induced by the optimal nonlinear contract. This choice of allowance results in a market-clearing resale price that is identical to the marginal rate of the optimal nonlinear contract. Subscribers seeking additional data purchase from those with unused allowances at the same cost they would have paid to the seller in a nonlinear contract. In anticipation of subscribers' resale revenue, the seller raises the price of the sharing contract to recoup the "lost" revenue from resales. Consequently, resale does not compromise the seller's revenue and performs comparably to the optimal nonlinear contract, which is recognized as an effective pricing strategy for managing heterogeneous demands (e.g., Oi 1971, Schmalense 1981, Rochet and Stole 2002).

Our work offers a new perspective on understanding the peer-to-peer resales and sheds new light on price discrimination. First, the sharing contract is a vehicle of price discrimination. Although it charges an *a priori* non-discriminatory price to all customers, peer-to-peer resale plays the discriminatory role of re-allocating the total supply to customers with heterogeneous demand. Second, our findings demonstrate the possibility of second-degree price discrimination even within a resale market. Conventional wisdom suggests that successful second-degree price discrimination, or equivalently nonlinear pricing, relies on preventing resale (e.g, Oi 1971, Wilson 1993), since the quantity discount nature of nonlinear pricing may cultivate arbitrage opportunities such that a customer buying a large order profit from breaking the order into several smaller lots and reselling them. However, our results suggest that the existence of a secondary market does not necessarily hinder the success of price discrimination. With price discrimination implemented through the sharing contract, resale becomes a necessary condition for success.

Despite the theoretical equivalence, sharing pricing and nonlinear pricing may not be used interchangeably in practice. The specific nature of the digital good has to be taken into account. Conventional nonlinear pricing schemes, such as two- and three-part tariffs, typically rely on the marginal rate to differentiate customers based on their demands. This approach is appropriate when a customer's actual consumption can be accurately measured, such as mobile data usage. Thus, nonlinear pricing may have an edge over sharing pricing due to the additional infrastructure cost of a trading platform. This is consistent with the CMHK's fade-away of its user trading platform.[3] However, it can be challenging for many digital goods to have a consensus on the user's consumption. Consider cloud file hosting services, where individual storage space usage may vary significantly as users add

[3] CMHK discontinued the platfrom in 2019.

or remove files. In such cases, adopting a two-part tariff may lead to ambiguity in defining actual usage—whether it's the maximum used space within a given time period, the average used space, or the space used at a specific time each month. This ambiguity arises because customer usage in cloud storage services may increase or decrease in a defined period of time, whereas the usage of other digital goods such as mobile data only increases. In light of this, sharing pricing emerges as an effective approach to monetizing from heterogeneous customers, particularly when their consumption levels may fluctuate either upward or downward in the measuring time frame, such as cloud storage services. This provides a theoretical explanation of Livedrive's long-lasting commitment to its reseller program since 2010.

The rest of the paper is organized as follows. Section 2 reviews relevant literature on nonlinear pricing and market resale. In Section 3, we use a simple example with two types of customer to illustrate the equivalence of sharing pricing and two-part tariffs and to elaborate the rationale of equivalence. A model with a general class of continuous customer heterogeneity is presented and solved in Section 4. We prove the equivalence of the continuous model and generalize the results in Section 5. Section 6 explores how consumer psychological costs may affect effectiveness of sharing and nonlinear contracts before making several concluding remarks in Section 7.

2. Related Literature

Our work is related to nonlinear pricing for price discrimination. In his influential work, Oi (1971) reveals that a two-part tariff not only is efficient in fulfilling customers' heterogeneous demands but also achieves a higher profit than a flat rate, which can be considered as a special case of the bucket pricing with an unlimited allowance. We refer to Wilson (1993) for a comprehensive overview of early studies in nonlinear pricing.

Recent studies on nonlinear pricing focus on the rise of three-part tariffs in telecommunication services and explore their advantages over two-part tariffs. First, the "free" allowance in a three-part tariff elevates customers' valuations of the service and thus increases the provider's revenue (Ascarza et al. 2012). Second, three-part tariffs are often more efficient than two-part tariffs thanks to the additional instrument—allowance. It has been shown that a small menu of three-part tariffs can be more profitable than a menu of two-part tariffs of any size (Bagh and Bhargava 2013). Moreover, a menu of three-part tariffs just performs as well as any nonlinear pricing mechanism when there are only two types of customers (Masuda and Whang 2006). Third, the fixed allowance in a three-part tariff allows the service provider to exploit customer demand uncertainty. Grubb (2009) proves that three-part tariff is an effective nonlinear pricing scheme for the service provider when customers exhibit a tendency to overestimate the precision of demand forecasts and hence underestimate the variance of future demand. Empirical work also esitmates the revenue gain a service provide may gain from consumers' biased expectations and inattention to usage (e.g., Grubb 2012, Grubb 2015, and Grubb and Osborne 2015). It also has been shown that demand uncertainty motivates customers to choose a three-part tariff with a larger allowance (Lambrecht et al. 2007) or a flat-rate unlimited plan (Lambrecht and Skiera 2006) even though a tariff with a smaller allowance indeed is a better choice. Yet, customers are able to learn their demand variations (Miravete 2002), despite some level of errors (Gopalakrishnan et al. 2015).

Although three-part tariffs are prevalent, it is challenging to characterize their optimal terms both in theory and empirically. Fibich et al. (2017) give out the first closed-form solution when customers are of two segments. Bhargava and Gangwar (2018) later propose a reformulation of the revenue-maximization problem for a general class of customer heterogeneity and structurally connect the optimal three-part tariffs and optimal two-part ones. We refer to Iyengar and Gupta (2009) for a detailed review on the empirical-driven difficulties of calculating optimal nonlinear pricing schemes.

Our paper contributes to the nonlinear pricing literature by describing a novel and non-discriminatory approach to price discrimination with a resale market—sharing pricing. We show that this novel mechanism is equivalent to nonlinear pricing in a frictionless resale marketand we establish the rationale for the equivalence. Huang et al. (2021) consider user trading that is similar to the sharing pricing. They rationalize the peer-to-peer sharing as a remedy for consumer overage disutility. In contrast, we demonstrate that peer-to-peer sharing can be an approach of price discrimination.

Peer-to-peer sharing also resembles the resale of used goods in a secondary market. Early studies believe that a secondary market may cannibalize the primary market (see, e.g., Levhari and Srinivasan 1969 and Rust 1986 for theoretical analyses and Hendel and Lizzeri 1999 for empirical evidence). Further reflection suggests that a secondary market may also increase primary demand, instead of cannibalizing it, if customers are strategic and anticipate a resale value (e.g., Hendel and Lizzeri 1999, Ishihara and Ching 2019). While firms can adjust their product upgrade frequencies to counteract peer-to-peer resale (e.g., Yin et al. 2010 for myopic customers and Guo and Chen 2018 for forward-looking customers), they may also take advantage of a secondary market to implement price discrimination (Anderson and Ginsburgh 1994) or coordinate channels (Desai et al. 2004, Shulman and Coughlan 2007). A fundamental difference between our paper and the literature on secondary markets of used goods is that the value of the resold goods does not depreciate in our setting. Therefore, customers' inter-temporal valuations do not play any role. Moreover, we consider a case where customers consume multiple units of the goods. The provider thus has one more instrument to leverage—the usage allowance. As we will show, this additional instrument helps establish the effect of price discrimination.

Secondary markets may also exist among retailers. When inventory is in shortage or excess, resale may occur among retailers to match supply with demand. This practice is often referred to as transshipment. One stream of research in transshipment concerns retailers' inventory stocking and allocation policies, e.g., Anupindi et al. (2001), Rudi et al. (2001), Granot and Sošić (2003), Sošić (2006), Çömez et al. (2012). The other stream investigates the impacts of transshipment on the profitability of supply chain members. Lee and Whang (2002) show that while transshipment among retailers improves market efficiency and benefits retailers, its impacts on the upstream manufacturer is indeterminate when the wholesale price is held exogenous. Dong and Rudi (2004) further point out that transshipment can also improve the manufacturer's profit if transshipment occurs among outlets of a chain store and the wholesale price is endogenously chosen. We refer to Paterson et al. (2011) for a detailed discussion on transshipment. Although the literature in this line of research is enormous, impacts of pricing schemes are usually of little interest since linear pricing, i.e., charging an exogenous wholesale price for each sold unit, is the convention in supply chains. However, our work captures the interaction between the provider's initial pricing choice and the underlying reselling dynamics and investigates the effectiveness of different pricing schemes for heterogeneous customers.

3. An Illustrating Example: The Case of Two Customer Types

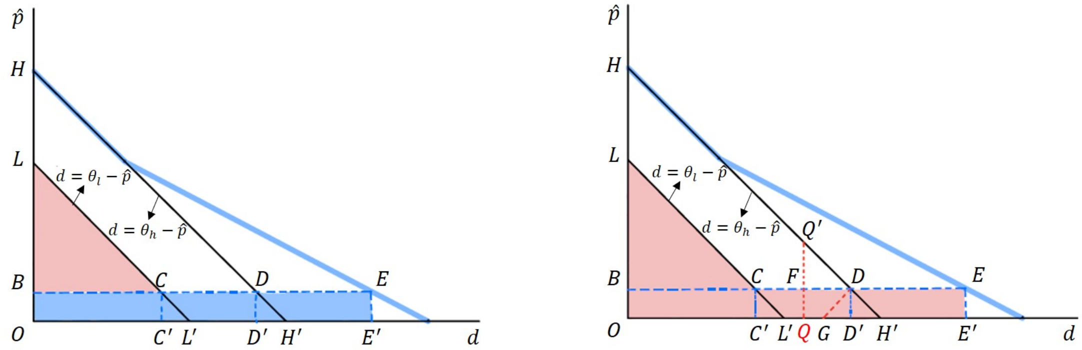
In this section, we illustrate graphically the equivalence of nonlinear pricing, in particular a two-part tariff, and peer-to-peer sharing pricing in a frictionless resale market. This simple two-type model not only visualizes the equivalence of the two contracts but also demonstrates the essential reason behind why such an equivalence holds—the one-to-one correspondence of marginal prices and aggregate demands. The same intuition applies when customer types follow a continuous distribution. However, the algebra is much more cumbersome; we discuss it in Section 5.

Suppose there are two customers: one customer is the high type with a valuation parameter θ_h; the other customer is the low type with a valuation parameter θ_l ($\theta_l < \theta_h$). The demands of the two customers are described in Figure 1(a) as $d_i = \theta_i - \hat{p}$, $i \quad \{l, h\}$, where $\hat{p}$ is the unit marginal price of the goods. The light blue bold line represents the aggregate demand of both customers as a function of $\hat{p}$. The provider cannot recognize the customers' types and offers a uniform contract to both. For digital goods, we assume that the marginal cost is zero, and it is optimal to serve both customers.[4]

We first review the design of an optimal nonlinear contract, i.e., a two-part tariff. For a given entry fee, points C, D, and E in Figure 1(a) exhibit the low-type, high-type, and aggregate demands at a

[4] It is indeed optimal to serve both customers if additional conditions on θ_l and θ_h are imposed.

Figure 1 Optimal Contracts: Two Types of Customers



(a) Two-Part Tariff (b) Sharing Contract

marginal price $\hat{p}$, respectively. The provider's total revenue from the marginal charge is thus equal to $A_{BEE\ O} = A_{BCC\ O} + A_{BDD\ O}$, where $A_{BCC\ O}$ and $A_{BDD\ O}$ denote the areas of the polygons and they represent the revenue contributions from the low- and high-type customers. The low and high types collect surpluses of A_{LCB} and A_{HDB}, respectively. Next, we turn to the choice of the entry fee. It is evident that in the case where both customers are served, the highest entry fee must be the one where the provider extracts all surplus from the low-type customer. Graphically, this means that the entry fee is equal to the area of triangle LCB, i.e., A_{LCB}. Consequently, the optimal two-part tariff maximizes the entry fee from both customers plus the revenue from marginal charges; i.e., it maximizes $2A_{LCB} + A_{BEE\ O}$. In the case of Figure 1(a), the optimal entry fee is $(3\theta_l - \theta_h)^2/8$, and the optimal marginal price is $(\theta_h - \theta_l)/2$, which leads to the low- and high-type demands $d_l = (3\theta_l - \theta_h)/2$ and $d_h = (\theta_l + \theta_h)/2$, respectively.

We next discuss the optimal sharing contract and elaborate its equivalence to a two-part tariff. The service provider still charges an entry fee but not a marginal price. Instead, the entry fee entitles customers to a usage allowance of Q units of the goods. Moreover, customers are allowed to trade their allowances freely on a resale market where their heterogeneous demands are fulfilled via a market clearing mechanism. The service provider has no direct control over the peer-to-peer sharing market, but she determines the entry fee and the associated allowance Q to maximize her revenue.

Figure 1(b) illustrates the design of a sharing contract. Without loss of generality, let us consider an allowance $Q \quad [\theta_l, \theta_h]$ (thus a total supplied allowance of $2Q$) and focus on the sharing market dynamics.[5] In this case, the low-type customer's utility-maximizing consumption level is equal to θ_l units, which results in $Q - \theta_l$ units of unused allowance. Meanwhile, the high-type customer uses up all of their allowances and is still interested in having more for a higher utility. The supply and demand sides of a resale market are thus formed. Now consider how the unit resale price, or equivalently

[5] It is straightforward to show that the optimal allowance must be between θ_l and θ_h.

the market-clearing price, arises. For the high type, his demand curve is still $d_h = \theta_h - \hat{p}$, which is represented by line $Q\,D$. The low type apparently has no problem with giving away $Q - \theta_l$ units for nothing. Yet he could be more strategic to sell more than $Q - \theta_l$ units. Note that when selling more than $Q - \theta_l$ units, the low-type customer's utility declines as his consumption drops. As a result, the reduction in his utility becomes the cost of his supply to the resale market, and they both change at the same rate. Specifically, $DD\,G$ depicts the supply cost and the symmetry of $DD\,G$ with CLC with respect to $Q\,Q$ reflects its equivalence to the reduction in low type's utility. Correspondingly, GD represents the low type's supply curve. Therefore, the intersection of the demand curve $Q\,D$ and the supply curve GD defines the market-clearing equilibrium at point D, whose vertical and horizontal coordinates represent the equilibrium market clearing price and the high type's demand, respectively. As a result, the utility of the high type equals to $A_{HQ\,QO} + A_{Q\,DF}$, where $A_{Q\,DF}$ is his utility gains from the peer-to-peer resale. For the low type, the horizontal coordinate of C corresponds to his effective usage allowance after resale. Therefore, the utility of the low type equals to $A_{LCC\,O} + A_{FDD\,Q}$, where $A_{FDD\,Q}$ is his utility gains from the peer-to-peer resale.

With an understanding of the sharing market, we now consider the choice of the entry fee. Again, we use Figure 1(b) for elaboration. As with the two-part tariff, the service provider sets the entry fee so that the low-type customer earns zero surpluses. Since the low-type customer earns a utility of $A_{LCC\,O} + A_{FDD\,Q}$ and $A_{FDD\,Q} = A_{CFQC}$ by symmetry, the entry fee for a given Q is thus equal to $A_{LCC\,O} + A_{CFQC}$, which can be rewritten as $A_{LCB} + A_{BFQO}$. Next, let us examine the effect of the allowance choice on the sharing contract. As Q increases, i.e., QQ shifts right, there are more supplies from the low type, and the market-clearing equilibrium D shifts downward along $Q\,D$. As a result, A_{LCB} and A_{BFQO} both vary as the allowance Q changes. The service provider thus sets the allowance Q to maximize her total revenue $2(A_{LCB} + A_{BFQO})$ from both types. Moreover, since the aggregate demand equals the total allowance $2Q$ under the market clearing mechanism, i.e., $BE = 2BF$, then $A_{BEE\,O} = 2A_{BFQO}$. Consequently, the optimal trading contract effectively maximizes $2A_{LCB} + A_{BEE\,O}$—the same objective as the optimal two-part tariff. Therefore, we have the equivalence of the two contracts. It can be further shown that the optimal allowance $Q \;= \theta_l$ and the resulting market clearing price equals $(\theta_h - \theta_l)/2$, which coincides with the optimal marginal price under the optimal two-part tariff. The low type consumes $d_l = (3\theta_l - \theta_h)/2$ units of allowance after selling $(\theta_h - \theta_l)/2$ units to the high type, who consumes $d_h = (\theta_l + \theta_h)/2$ units in total with $(\theta_h - \theta_l)/2$ units purchased from the resale market.

Although the nonlinear two-part tariff and the sharing contract are implemented in seemingly distinct ways—one offering discounts for large volume consumption and one charging a uniform price to all, they are in fact equivalent. Figures 1(a) and 1(b) demonstrate this equivalence by geometrically showing the two pricing schemes' identical revenue-maximizing objectives, which can be achieved by choosing the position of E on the aggregate demand curve. The one-to-one correspondence between the marginal price and the aggregate demand ensures that only one of them needs to be maneuvered for optimality. As shown in Figure 1(a), the nonlinear two-part tariff controls the position of E by setting the marginal price, which leads to the optimal aggregate demand and is also directly used to differentiate customers according to their heterogeneous demands. In contrast, the sharing contract in Figure 1(b) adopts another approach: it controls the total supply, and thus the aggregate demand, by consciously setting the allowance and relies on the peer-to-peer resale to fulfill customer heterogeneous demands. In sum, the one-to-one correspondence between the marginal price and the aggregate demand paves the way for the equivalence.

In comparison with the nonlinear two-part tariff, the sharing contract imposes a non-discriminatory price for all customers despite their heterogeneity. Moreover, the sharing contract does not prohibit a resale market. In fact, it takes advantage of resale to discriminate in meeting heterogeneous demands. However, its efficiency may depend on the potential market friction in the peer-to-peer resale process—an issue nonexistent for a nonlinear two-part tariff.

In the rest of the paper, we generalize and extend the equivalence of the nonlinear contract and the sharing contract to the case where customer types follow a continuous distribution. We show that the key insight and intuition remain true with more realistic assumptions.

4. Model Description and Optimal Contract Design

In this section, we will first layout the micro-model of customer consumption decisions and then characterize the optimal contracts. Specifically, we will consider the optimal terms of the sharing contact and the nonlinear contract. We also provide the analysis of the bucket contract in EC1.

4.1. Customer Profile

Consider a monopoly (she) that offers a service contract to potential customers. We assume that the marginal variable cost of the service is zero once the facility and infrastructure are established and the maintenance cost is approximately fixed. This assumption is often valid for digital goods, such as e-books, cellular data and cloud storage (e.g., Essegaier et al. 2002, Sundararajan 2004, Bhargava and Gangwar 2018).

Without loss of generality, we normalize the total market size to one (e.g., Essegaier et al. 2002, Bhargava and Gangwar 2018) and assume customers are infinitesimal relative to the market size. Customers have heterogeneous valuations for the goods. For each unit consumed, a type-θ customer (he) receives a reward of θ, which follows a continuous and differentiable distribution $F(\cdot)$ with an increasing failure rate (IFR) on $[0, \Theta]$. For tractability, we also need $F(\cdot)$ to have the structural property indicated in Assumption 1. Let us define two ancillary functions from the customer valuation distribution $F(x)$. Denote $G(\cdot)$ and $g(\cdot)$ as

$$G(x) = \frac{\int_0^x \bar{F}(t)\mathrm{d}t}{\int_0^\Theta \bar{F}(t)\mathrm{d}t} \text{ and } g(x) = \frac{dG(x)}{dx} = \frac{\bar{F}(x)}{\int_0^\Theta \bar{F}(t)\mathrm{d}t},$$

where $x \in [0, \Theta]$ and $\bar{F}(x) = 1 - F(x)$. Note that $G(x)$ and $g(x)$ can also be interpreted as a cumulative distribution function and its probability density function, respectively. Therefore, we can also define

$$\frac{g(x)}{\bar{G}(x)} = \frac{\bar{F}(x)}{\int_x^\Theta \bar{F}(t)\mathrm{d}t}$$

as the failure rate of $G(\cdot)$.

Assumption 1. *$G(\cdot)$ has an increasing failure rate (IFR), i.e., $g(x)/\bar{G}(x)$ increases in x, and its failure rate is no less than that of $F(\cdot)$, i.e., $g(x)/\bar{G}(x) \geq f(x)/\bar{F}(x)$ for any $x \in [0, \Theta]$.*

Assumption 1 is relatively mild. Many distributions with IFRs in the literature (e.g., uniform, normal, and exponential) induce $G(\cdot)$'s that satisfy Assumption 1. In Sections 4.2 and 4.3, specifically Propositions 1 and 4, Assumption 1 allows us to derive analytical pricing expressions for general heterogeneous distributions under the sharing and nonlinear contracts.

We define the utility received by a type-θ customer as

$$u(d \mid \theta) = \theta d - \frac{1}{2}d^2, \tag{1}$$

where d is the demand for the goods. The quadratic term $d^2/2$ implies a diminishing marginal value of per-unit consumption. For example, health hazard from phone use is expected to increse with the time a customer spends on the phone, leading to a decreasing net marginal return. We acknowledge that a quadratic cost function might be restrictive, but it is helpful in establishing useful insights (see, e.g., Rochet and Stole 2002 and Desai et al. 2018 for analytical modeling and Lambrecht et al. 2007 for empirical work).

Let $c(d \mid \theta)$ be a type-θ consumer's total service fee paid to the provider. All customers simultaneously decide whether to subscribe.[6] A customer's consumption $d(\theta)$ maximizes the net surplus

$$s(d \mid \theta) = u(d \mid \theta) - c(d \mid \theta) = \theta d - \frac{1}{2}d^2 - c(d \mid \theta). \tag{2}$$

Note that the total cost $c(d \mid \theta)$ depends on the terms of the contract and its corresponding fee structure. Specifically, we compare two contracts: (i) a sharing contract, in which the provider charges

[6] For simplicity, we assume that the customers choose to subscribe when they are indifferent.

p_s for allowance up to Q_s units, but allows subscribing customers to resell unused goods to one another; and (ii) a nonlinear contract, in which the provider charges p_n for a consumption allowance up to Q_n units and a unit overage price $\hat{p}_n$ for any consumption in excess of the allowance. In particular, when $Q_n = (>)0$, the nonlinear contract is a two(three)-part tariff.

The provider offers a contract without observing the type of a consumer. Hence, she is not able to implement perfect price discrimination. However, the provider has full information of the customer type distribution $F(\cdot)$ and takes it into account when designing the optimal contract terms.

We characterize the optimal sharing and nonlinear contracts in Sections 4.2 and 4.3, respectively.

4.2. Sharing Pricing

In this subsection, we consider the sharing contract, which resembles bucket pricing by offering heterogeneous customers a uniform contract with an allowance Q_s at a price p_s but differs from it by allowing peer-to-peer resale via a sharing platform established by the service provider. We allow the service provider to charge a unit allowance transaction fee t_s from both parties of a resale. However, as we will show later, the provider has no incentive to impose a positive t_s at the optimality.[7] Following the convention in the literature of resale (e.g., Lee and Whang 2002, Yin et al. 2010), the equilibrium sharing price, denoted as $\hat{p}_s$, is assumed to arise via a market clearing mechanism and the peer-to-peer sharing market is efficient with no friction other than the potential transaction fee t_s.

Let $d_s(\theta)$ represent a type-θ customer's demand. Then, we can write his total cost as

$$c_s(d_s \mid \theta) = p_s + \hat{p}_s \cdot (d_s(\theta) - Q_s) + t_s \cdot |d_s(\theta) - Q_s|, \tag{3}$$

which includes the payment to the service provider p_s, the cost or revenue from resale, and the transaction fee in the sharing process. Conditional on the fact that this type-θ customer subscribes to the sharing contract, he purchases more of the goods in the sharing market if $d_s(\theta) - Q_s > 0$ and incurs a higher cost than p_s. Otherwise, $d_s(\theta) - Q_s < 0$ and he sells some of his allowance in the sharing market and thus profits in the sharing resale market, which results in a lower cost than p_s. Given the cost function in (3), the surplus of a type-θ customer can be expressed as

$$s_s(d_s \mid \theta) = u(d_s \mid \theta) - c_s(d_s \mid \theta) = \theta d_s - \frac{1}{2}d_s^2 - p_s - \hat{p}_s(d_s - Q_s) - t_s \cdot |d_s(\theta) - Q_s| \tag{4}$$

and she solves $\max_{d_s \geq 0}\ s_s(d_s \mid \theta)$ for the optimal demand $d_s(\theta)$ and subscribes if $s_s(d_s \mid \theta) \geq 0$.

To avoid the trivial case where no one is in need of extra goods, i.e., $d_s(\theta) \leq Q_s$ for any $\theta \quad [0, \Theta]$, we make the following mild and plausible assumption when considering the sharing contract.

Assumption 2. *The allowance Q_s in the sharing contract is no more than the maximum total demand of all customers; i.e., $Q_s \leq \mathsf{E}[\theta] = \int_0^{\Theta} \theta f(\theta) d\theta$.*

[7] This result still holds even if the transaction fee is unilateral, i.e., paid by either the buyer or the seller.

In the absence of any cost, the utility function in (1) implies that a type-θ customer's demand equals θ. Therefore, the potential total demand from all customers is no more than $\int_0^{\Theta} \theta f(\theta) d\theta$. It is evident that no one needs additional allowance if $Q_s > \int_0^{\Theta} \theta f(\theta) d\theta$.

PROPOSITION 1. *If Assumption 1 holds, there exists an optimal sharing contract with* $t_s = 0$.

Proposition 1 ascertains that it is in the provider's best interest to eliminate frictions in the sharing market—a strictly positive transaction fee t_s restrains the provider's pricing power of setting a high initial price p_s. As the transaction fee t_s diminishes, reselling becomes more seamless. On the one hand, low-type customers could take advantage of reselling to earn a greater surplus. On the other hand, high-type customers could purchase more of the goods and enjoy more. Since sellers and buyers both benefit from a lower transaction fee, the service provider can improve her revenue by selling a more expensive sharing contract in the first place. Assumption 1 implicitly ensures that raising the sharing contract price is always more effective than imposing a transaction fee.

Proposition 1 has a significant managerial implication: the traceability of digital goods is not essential to maneuver the impact of resales. At the optimal solution, the seller is not necessarily taking a cut from resale and only needs to profit from sales in the primary market. In other words, although our model assumes the peer-to-peer sharing market is curated by the seller, our results stay intact as long as resales take place in a free market (rather than in a market managed by a profit-maximizing third party).

Proposition 1 facilitates the characterization of the optimal sharing contract. Without loss of generality, we restrict our attention to a frictionless sharing market and omit the transaction fee in the rest of the paper. The next lemma characterizes customers' subscription and consumption decisions in this frictionless sharing market.

LEMMA 1. *Assume that the service provider offers a sharing contract* (p_s, Q_s). *There exists a unique* $\bar{\theta}_s$ *such that customers subscribe to the service if and only if* $\theta \geq \bar{\theta}_s$. *The subscriber's demand satisfies* $d_s(\theta) = \max\{\theta - \hat{p}_s, 0\}$, *where* $\hat{p}_s$ *is the market clearing price of a unit of the good.*

Lemma 1 first shows that customers' subscription decisions retain a threshold structure: only those who value each unit of the goods more than $\bar{\theta}_s$ subscribe. Moreover, Lemma 1 pinpoints the dependence of each type's optimal consumption level on the market-clearing price.

It is worth mentioning that the sharing process may enable arbitrage opportunities and speculative resale: some customers may subscribe but not use any of the allowances and sell it all to others. However, it is challenging to characterize explicitly when speculators may exist in equilibrium or even demonstrate the existence of a resale market equilibrium. The difficulties arise from the endogeneity and nonlinearity of the supply and the demand in the resale market with respect to the contract parameters. To overcome the technical obstacles, we first explore the structural properties of an equilibrium. Using these properties, we construct the candidate equilibria with and without speculators and then demonstrate their validity and uniqueness. The next lemma summarizes the structural properties of the emerging market-clearing equilibria that may or may not involve speculators.

LEMMA 2. *Assume that the service provider offers a sharing contract* (p_s, Q_s). *Let* $\bar{\theta}_s$ *represent the cutoff so that customers of type* $\theta \geq \bar{\theta}_s$ *subscribe and* $\hat{p}_s$ *be the equilibrium market clearing price.*

(i) The sharing market has an equilibrium with speculators; i.e., some subscribers resell all their allowances at a unique market clearing price $\hat{p}_s \geq 0$ *if and only if* $\hat{p}_s \geq p_s/Q_s$ *and* $0 \leq \bar{\theta}_s < \hat{p}_s$. *Moreover,* $\hat{p}_s$ *and* $\bar{\theta}_s$ *must satisfy the following equality*

$$Q_s \bar{F}(\bar{\theta}_s) + \hat{p}_s \bar{F}(\hat{p}_s) = \int_{\hat{p}_s}^{\Theta} \theta f(\theta)\, d\theta. \tag{5}$$

(ii) The sharing market has an equilibrium without speculators; i.e., all subscribers consume some of the allowance and there is a unique market clearing price $\hat{p}_s \geq 0$, *if and only if* $0 \leq \hat{p}_s \leq p_s/Q_s$ *and* $\bar{\theta}_s \geq \hat{p}_s$. *Moreover,* $\hat{p}_s$ *and* $\bar{\theta}_s$ *must satisfy the following equality*

$$Q_s \bar{F}(\bar{\theta}_s) + \hat{p}_s \bar{F}(\bar{\theta}_s) = \int_{\bar{\theta}_s}^{\Theta} \theta f(\theta)\, d\theta. \tag{6}$$

Lemma 2(i) quantifies two necessary conditions for the existence of speculators, i.e., those subscribing to the service but resell all their allowances. First, speculators are present only if the market-clearing price is no less than the average unit price derived from the sharing contract, i.e., if $\hat{p}_s \geq p_s/Q_s$. Otherwise, reselling is not profitable at all for speculators. Second, the fact that speculators do not consume any of the goods also suggests that their intrinsic valuations θ's are less than the per-unit payoff from reselling; i.e., $\bar{\theta}_s \leq \theta < \hat{p}_s$. Otherwise, consuming some of the goods contributes more to their net surplus than selling them in the sharing market. In contrast, Lemma 2(ii) depicts the complementary case that there are no speculators.

Lemma 2 shows possible equilibria under sharing but does not specify when a specific equilibrium arises. The next proposition provides a sufficient and necessary condition for a sharing equilibrium.

PROPOSITION 2. *Assume that the service provider offers a sharing contract* (p_s, Q_s).

(i) The sharing market has a unique equilibrium with speculators if and only if $0 \leq Q_s < \overline{Q}_s(p_s, Q_s)$;

(ii) The sharing market has a unique equilibrium without speculators if and only if $Q_s \geq \overline{Q}_s(p_s, Q_s)$ *and* $p_s/Q_s + Q_s/2 \leq \Theta$,

where $\overline{Q}_s(p_s, Q_s) = \int_{p_s/Q_s}^{\Theta} \theta f(\theta)\, d\theta / \bar{F}(p_s/Q_s) - p_s/Q_s$.

Proposition 2 structurally connects a sharing contract (p_s, Q_s) with a possible equilibrium via $\overline{Q}_s(p_s, Q_s)$. We acknowledge that $\overline{Q}_s(p_s, Q_s)$ is homogeneous of degree 0, i.e., $\overline{Q}_s(p_s, Q_s) = \overline{Q}_s(kp_s, kQ_s)$ for a scalar k. This property is profound in characterizing the optimal sharing contract. We next show which of the two forms of equilibria, sharing with or without speculators, may yield a higher revenue for the provider.

According to Lemma 1, customers' subscription decisions have a threshold structure; we thus write the provider's revenue-maximizing problem as

$$\max_{p_s, Q_s} \Pi_s(p_s, Q_s) = p_s \cdot \bar{F}(\bar{\theta}_s(p_s, Q_s)) \quad \text{s.t. } p_s \geq 0 \text{ and } Q_s \geq 0, \tag{7}$$

where $\bar{\theta}_s(p_s, Q_s)$ is the cutoff for customer subscriptions as identified in Lemma 1.

Proposition 3 **(Optimal Sharing Contract)**. *It is optimal to offer a sharing contract so that there are no speculators subscribing in equilibrium. Under such an optimal sharing contract,*

(i) the provider offers an allowance of Q_s units, where Q_s is the solution to

$$\frac{\int_{Q_s}^{\Theta} \theta f(\theta)\, d\theta}{2\bar{F}(Q_s)} = \frac{\bar{F}(Q_s)}{f(Q_s)}, \tag{8}$$

and charges $p_s = \dfrac{{Q_s}^2}{2} + \dfrac{1}{2}\left(\dfrac{2\bar{F}(Q_s)}{f(Q_s)} - Q_s\right)^2$;

(ii) the resulting equilibrium market clearing price $\hat{p}_s = \sqrt{2p_s - {Q_s}^2}$;

(iii) customers subscribe to the service if and only if $\theta \geq \bar{\theta}_s = Q_s$ and $d_s(\theta) = \theta - \hat{p}_s \geq 0$.

Proposition 3 first confirms that a sharing market with speculators is not financially desirable for the service provider. All earnings that speculators are able to pocket are losses of revenue that the service provider could otherwise obtain. The service provider can effectively price out speculators by offering a large allowance and indirectly inducing an unprofitable market clearing price for speculators. Specifically, let us assume that the service provider offers a sharing contract (p_s, Q_s) that results in a sharing market with speculators in equilibrium. By definition, speculators do not consume any of the goods but only resell them. Therefore, for speculators of type θ,

$$s_s(d_s = 0 \mid \theta) = u(d_s = 0 \mid \theta) - c_s(d_s = 0 \mid \theta) = \hat{p}_s Q_s - p_s \geq 0,$$

where $\hat{p}_s$ is the market clearing price under (p_s, Q_s).

We now construct a sharing contract based on (p_s, Q_s) to induce a sharing market without speculators. Let us scale up the price and the allowance of the sharing contract simultaneously by a multiplier of $k \geq 1$. The speculators' surplus under the new sharing contract (kp_s, kQ_s) can be written as $s_s(d_s = 0 \mid \theta) = \hat{p}_s kQ_s - kp_s$, where $\hat{p}_s$ is the new market-clearing price. Since the service provider offers a larger allowance to all subscribers, this, on one hand, increases potential supplies on the sharing market. On the other hand, it also reduces potential demands on the sharing market. Thus, the market-clearing price $\hat{p}_s$ decreases as k scales up and so does $s_s(d_s = 0 \mid \theta) = \hat{p}_s k Q_s - k p_s$. At the point k is large enough such that $kQ_s \geq \overline{Q}_s(kp_s, kQ_s) = \overline{Q}_s(p_s, Q_s)$, Proposition 2(ii) implies that speculators do not exist any more. Although eliminating speculators reduces the number of subscribers, the increased contract price is more than enough to offset the loss and it increases revenues.

The next corollary identifies buyers and sellers in the peer-to-peer sharing market.

COROLLARY 1. *When the sharing market reaches an equilibrium, subscribers of type* $\theta \quad (Q_s + \hat{p}_s, \Theta]$ *buy extra goods from subscribers of type* $\theta \quad (\bar{\theta}_s, Q_s + \hat{p}_s)$.

4.3. Nonlinear Pricing

In this subsection, we consider the nonlinear contract. Like the bucket and sharing contracts, the nonlinear contract also specifies a base price p_n for an allowance up to Q_n. In contrast to the bucket contract, a nonlinear contract allows subscribers to go beyond the allowance at a unit overage price $\hat{p}_n$. The nonlinear contract we consider is often referred to as a three-part tariff since it is defined by three contract parameters $(p_n, Q_n, \hat{p}_n)$. The most common form of nonlinear contracts, two-part tariffs, is a special case of three-part tariffs when $Q_n = 0$. For a given nonlinear contract $(p_n, Q_n, \hat{p}_n)$, a type-θ subscriber who uses d_n units of the goods incurs a cost of

$$c_n(d_n \mid \theta) = p_n + \hat{p}_n \cdot (d_n - Q_n)^+.$$

For given p_n, Q_n, and $\hat{p}_n$, a subscriber solves the following problem

$$\max_{d_n \geq 0} \; s_n(d_n \mid \theta) = u(d_n \mid \theta) - c_n(d_n \mid \theta) = \theta d_n - \frac{1}{2}d_n^2 - p_n - \hat{p}_n(d_n - Q_n)^+ \tag{9}$$

for her optimal demand, which is summarized below.

LEMMA 3. *Assume that the service provider offers a nonlinear contract* $(p_n, Q_n, \hat{p}_n)$. *If a type-*θ *customer subscribes, her optimal consumption level satisfies*

$$d_n(\theta) = \begin{cases} \theta, & \text{if } 0 \leq \theta < Q_n \\ Q_n, & \text{if } Q_n \leq \theta < \hat{p}_n + Q_n \\ \theta - \hat{p}_n, & \text{otherwise.} \end{cases} \tag{10}$$

Note that Lemma 3 characterizes customer demands only if they do subscribe to the service, but it does not say who subscribes. Our next result identifies the subscribers.

LEMMA 4. *Assume that the service provider offers a nonlinear contract* $(p_n, Q_n, \hat{p}_n)$. *A nonzero fraction of customers subscribe if and only if* $p_n \leq \frac{1}{2}(\Theta - \hat{p}_n)^2 + \hat{p}_n Q_n$. *Specifically, there exists a unique* $\bar{\theta}_n$ *such that customers subscribe to the service if and only if* $\theta \geq \bar{\theta}_n$, *where*

$$\bar{\theta}_n = \begin{cases} \sqrt{2p_n}, & \text{if } 0 \leq p_n < Q_n^2/2 \\ p_n/Q_n + Q_n/2, & \text{if } Q_n^2/2 \leq p_n < \hat{p}_n Q_n + Q_n^2/2 \\ \hat{p}_n + \sqrt{2(p_n - \hat{p}_n Q_n)}, & \text{if } \hat{p}_n Q_n + Q_n^2/2 \leq p_n \leq \frac{1}{2}(\Theta - \hat{p}_n)^2 + \hat{p}_n Q_n. \end{cases} \tag{11}$$

By Lemma 4, we can write the provider's revenue-maximizing problem as follows

$$\max_{p_n, Q_n, \hat{p}_n} \quad \Pi_n(p_n, Q_n, \hat{p}_n) = p_n \bar{F}(\bar{\theta}_n) + \hat{p}_n \int_{\max\{\bar{\theta}_n, \hat{p}_n + Q_n\}}^{\Theta} [(\theta - \hat{p}_n) - Q_n] f(\theta) \mathrm{d}\theta \tag{12}$$
$$\text{s.t.} \qquad p_n \geq 0, \; Q_n \geq 0, \; \text{and } \hat{p}_n \geq 0,$$

where $\bar{\theta}_n$ represents the fraction of customers who subscribe to the service as identified in (11). The first term $p_n \bar{F}(\bar{\theta}_n)$ of $\Pi_n(p_n, Q_n, \hat{p}_n)$ accounts for the revenue from service subscriptions and the second term $\hat{p}_n \int [(\theta - \hat{p}_n) - Q_n] \mathrm{d}F(\theta)$ captures the provider's revenue from overage surcharges. The lower limit $\max\{\bar{\theta}_n, \hat{p}_n + Q_n\}$ of the integration specifies the subscribers who use more than the allowance Q_n units. If $\bar{\theta}_n \leq \hat{p}_n + Q_n$, by (10) only those of $\theta \geq \hat{p}_n + Q_n$ pay for overage; otherwise, all subscribers consume no less than the allowance.

It is difficult to derive a closed-form solution to the revenue-maximization problem in (12). Both Fibich et al. (2017) and Bhargava and Gangwar (2018) show that the revenue surface is highly non-linear and non-differentiable with potentially multiple local optima. Fibich et al. (2017) characterize the closed-form solutions for markets with homogeneous customers or only two types. Bhargava and Gangwar (2018) propose a reformulation for a general case of heterogeneous customers and apply it to obtain analytical solutions when the heterogeneity follows some specific distributions. We take a constructive approach by imposing Assumption 1—a slightly more restrictive condition than that in Bhargava and Gangwar 2018 and employing the duality theory. Our approach first identifies the only possible interval in which the optimal solution can lie. We then eliminate the possibility that this optimality occurs at an interior point of the interval and thus confine the optimal solution to the boundary. We next explicitly characterize the solution to (12) for general customer heterogeneity.

PROPOSITION 4 **(Optimal Nonlinear Contract)**. *Suppose that Assumption 1 holds. Under the optimal nonlinear contract,*

(i) customers subscribe to the service if and only if her type $\theta \geq \bar{\theta}_n$, where $\bar{\theta}_n$ is the solution to

$$\frac{\int_{\bar{\theta}_n}^{\Theta} \theta f(\theta) d\theta}{2\bar{F}(\bar{\theta}_n)} = \frac{\bar{F}(\bar{\theta}_n)}{f(\bar{\theta}_n)};$$

(ii) the optimal price p_n and the optimal allowance Q_n are all solutions to

$$\frac{(\bar{\theta}_n - \hat{p}_n)^2}{2} = p_n - \hat{p}_n Q_n \; \text{ s.t. } p_n \geq 0 \text{ and } 0 \leq Q_n \leq \bar{\theta}_n - \hat{p}_n, \tag{13}$$

where $\hat{p}_n = \int_{\bar{\theta}_n}^{\Theta} \theta f(\theta) d\theta / \bar{F}(\bar{\theta}_n) - \bar{\theta}_n$ is the optimal overage rate.

As with the sharing contract, the subscription decision under the nonlinear contract also follows a threshold structure.

Although the overage rate $\hat{p}_n$ is unique, there are multiple optimal base prices p_n's and allowances Q_n's as defined in (13). Moreover, it is easy to find a solution to (13) with $Q_n = 0$. This particular solution is in fact a two-part tariff, where p_n is the base price and $\hat{p}_n$ is the marginal price. This result resonates with the finding in Bhargava and Gangwar (2018), which provides a general condition under which three-part tariffs yield the same revenue as two-part ones. Although a more restrictive condition, i.e., Assumption 1, is imposed to establish our result, the condition allows us to derive a sharper expression for general heterogeneous distributions.

An important property of the optimal nonlinear contract is that the service provider offers a relatively small allowance Q_n such that all subscribers consume more than the contracted allowance and pay the overage surcharge.

Corollary 2. *Under the optimal nonlinear contract, $\theta_n \geq \hat{p}_n + Q_n$ and all subscribers consume more than the allowance Q_n, i.e., $d_n(\theta) = \theta - \hat{p}_n \geq Q_n$ for $\theta \quad [\bar{\theta}_n, \Theta]$.*

Intuitively, a small allowance implies a relatively low base price p_n, which makes the service more affordable and helps to increase the market coverage. The provider is compensated by selling extra goods to subscribers in need and achieves second-degree price discrimination: subscribers who demand more pay lower average unit prices. Obviously, the implementation of price discrimination allows the provider to be able to better serve customers' heterogeneous demands.

5. Comparing Sharing and Nonlinear Contracts

We characterize the optimal terms of the sharing contract and the nonlinear contract in Section 4. In this section, we will compare these two contracts in market coverage, customer consumption, and provider revenue.[8]

5.1. Equivalence of Sharing and Nonlinear Contracts

It may seem that the sharing contract is not as effective as the nonlinear contract because its uniform terms prevent it from using direct price discrimination. Moreover, the sharing contract does not allow the peer-to-peer sharing process to be controlled either. However, we will show that despite these unfavorable features, the sharing contract performs just as well as a price discriminatory nonlinear contract: both contracts attract the same number of subscribers, lead to the same individual surplus, and result in the same provider revenue. Despite its uniformity to all subscribers, the sharing contract in fact is a method of price discrimination with a resale market. Theorem 1 states the equivalence from the perspective of the service provider.

[8] We also compare the sharing and nonlinear contracts with the bucket contract in EC1.2. The main conclusion is that both are more effective in managing heterogeneous demands than the bucket contract as the bucket contract charges a uniform price for the same allowance to all customers.

THEOREM 1 **(Revenue Equivalence of Sharing and Nonlinear Contracts)**. *Comparing the optimal sharing and nonlinear contracts, we find that*

(i) both yield the same revenue for the provider, i.e., $\Pi_s(p_s, Q_s) = \Pi_n(p_n, Q_n, \hat{p}_n)$;

(ii) both result in the same market coverage, i.e., $\bar{\theta}_s = \bar{\theta}_n$.

In order to explain the revenue equivalence of the two contracts, we illustrate how an equivalent sharing contract can be constructed from an optimal nonlinear contract $(p_n, Q_n, \hat{p}_n)$. The seemingly desirable feature of the nonlinear contract is the capability of allowing heterogeneous customers to consume different amounts of goods according to their types. Although the uniformity of the sharing contract makes it impossible to directly distinguish among customers of various potential demands, peer-to-peer resale offers an alternative: subscribers who do not need all their allowances have opportunities to resell their unused quota to those who need them. Yet, it is still tricky since the provider cannot directly intervene in subscribers' peer-to-peer resale and she also has little control over the market-clearing price and the customers' demands.

Nevertheless, we argue that a meticulously chosen allowance in the sharing contract has the same discriminatory function as the optimal nonlinear contract. Specifically, the provider should set the sharing contract's allowance so that the total provision of the goods under the contract equals the total supply of the goods under the optimal nonlinear contract. Mathematically, this means

$$\underbrace{Q_s \bar{F}(\bar{\theta}_s)}_{\text{total supply under sharing contract}} = \underbrace{\overbrace{Q_n \bar{F}(\bar{\theta}_n)}^{\text{supply via allowance}} + \overbrace{\int_{\bar{\theta}_n}^{\Theta} [d_n(\theta) - Q_n]^+ \, \mathrm{d}F(\theta)}^{\text{supply via overage}}}_{\text{total supply under nonlinear contract}},$$

which can be further written as

$$Q_s \bar{F}(\bar{\theta}_s) = Q_n \bar{F}(\bar{\theta}_n) + \int_{\bar{\theta}_n}^{\Theta} [(\theta - \hat{p}_n) - Q_n] \mathrm{d}F(\theta) \tag{14}$$

since Corollary 2 shows that $d_n(\theta) = \theta - \hat{p}_n \geq Q_n$ for $\theta \quad [\bar{\theta}_n, \Theta]$. To facilitate our discussion, let us for now assume that the two contracts induce an equal market coverage, i.e., $\bar{\theta}_s = \bar{\theta}_n = \bar{\theta}$. This assumption does not necessarily hold. However, we will demonstrate its validity at the end of this section. Under the equal market coverage assumption, (14) can be written as

$$Q_s = Q_n + \int_{\bar{\theta}}^{\Theta} [(\theta - \hat{p}_n) - Q_n] \mathrm{d}F(\theta) \Big/ \bar{F}(\bar{\theta}). \tag{15}$$

Eq. (15) has several important implications. The first and also intuitive one is that an equivalent sharing contract has to offer a larger allowance than its counterpart under the optimal nonlinear contract; i.e., $Q_s \geq Q_n$.

Second, (15) implies that the market-clearing price $\hat{p}_s$ equals the per-unit overage rate $\hat{p}_n$ under the optimal nonlinear contract. To see this, we need to establish a one-to-one correspondence between the market-clearing price $\hat{p}_s$ and the sharing contract allowance Q_s. It can be shown that there is no speculator in the peer-to-peer trading process if the sharing contract attempts to replicate the performance of the optimal nonlinear contract.

Hence, by Lemma 1, $d_s(\theta) = \theta - \hat{p}_s \geq 0$ for any $\theta \geq \bar{\theta}$.[9] Since the total allowance provision equals the total consumption under the sharing contract, we have

$$Q_s \bar{F}(\bar{\theta}) = \int_{\bar{\theta}}^{\Theta} (\theta - \hat{p}_s)\, dF(\theta) = \int_{\bar{\theta}}^{\Theta} \theta dF(\theta) - \hat{p}_s \bar{F}(\bar{\theta}), \tag{16}$$

which evidently shows a one-to-one correspondence between Q_s and $\hat{p}_s$. Such a correspondence suggests that the allowance Q_s is an instrument for manipulating the market clearing price $\hat{p}_s$. The choice of Q_s thus plays a role in distinguishing heterogeneous subscribers in their consumption decisions $d_s(\theta)$'s—even if it appears to be irrelevant. We further notice that the effectiveness of making use of the allowance Q_s for consumption control is identical to that of the overage rate $\hat{p}_n$ in the nonlinear contract. This can be seen from the following supply-demand equality of the nonlinear contract,

$$\underbrace{Q_n \bar{F}(\bar{\theta}) + \int_{\bar{\theta}}^{\Theta} [(\theta - \hat{p}_n) - Q_n] \mathrm{d}F(\theta)}_{\text{total supply under nonlinear contract}} = \underbrace{\int_{\bar{\theta}}^{\Theta} (\theta - \hat{p}_n)\, dF(\theta)}_{\text{total demand under nonlinear contract}} = \int_{\bar{\theta}}^{\Theta} \theta dF(\theta) - \hat{p}_n \bar{F}(\bar{\theta}). \tag{17}$$

Together with (15) and (16), (17) indicates $\hat{p}_s = \hat{p}_n$. Consequently, we claim

$$d_s(\theta) = \theta - \hat{p}_s = \theta - \hat{p}_n = d_n(\theta), \tag{18}$$

which reveals that the market clearing price $\hat{p}_s$ has the same discriminatory effect on subscribers' demands as the nonlinear contract's overage rate $\hat{p}_n$ and so does the allowance Q_s.

Last but not least, (15) reveals that the extra allowance $Q_s - Q_n$ equals the average overage consumption of all nonlinear contract subscribers. Therefore, if the provider intends to obtain the same revenue through the sharing contract, she has to compensate herself for the revenue loss from the provision of these additional $Q_s - Q_n$ units of allowance, which costs a subscriber $\hat{p}_n(Q_s - Q_n)$ under the nonlinear contract. In other words, if the provider sets the sharing contract's price at $p_s = p_n + \hat{p}_n(Q_s - Q_n)$, then

$$\begin{aligned}
\Pi_s(p_s, Q_s) &\overset{(7)}{=} p_s \cdot \bar{F}(\bar{\theta}) \\
&= (p_n + \hat{p}_n(Q_s - Q_n))\bar{F}(\bar{\theta}) \\
&\overset{(14)}{=} p_n \bar{F}(\bar{\theta}) + \hat{p}_n \int_{\bar{\theta}}^{\Theta} [(\theta - \hat{p}_n) - Q_n] \mathrm{d}F(\theta) \\
&\overset{(12)}{=} \Pi_n(p_n, Q_n, \hat{p}_n),
\end{aligned}$$

i.e., she collects the same revenue as she would have under the optimal nonlinear contract.

[9] The equality occurs only at $\theta = \bar{\theta}$.

Conversely, a nonlinear contract can be constructed to replicate an optimal sharing contract. The revenue equivalence of the two respective contracts is thus established as presented in Theorem 1(i). Moreover, Proposition 5 summarizes the quantitative connections of the two contracts' parameters, which we have elaborated on above.

Proposition 5. *Comparing the optimal sharing and nonlinear contracts, we have that*

(i) $p_s > p_n$ *and* $Q_s > Q_n$*;*

(ii) $\hat{p}_s = \hat{p}_n$ *and* $p_s = p_n + \hat{p}_s(Q_s - Q_n)$*;*

(iii) $\hat{p}_n \leq p_s/Q_s \leq p_n/Q_n$*.*

We note that $\hat{p}_s = \hat{p}_n$ in Proposition 5ii may be reminiscent of the well-known equivalence of carbon tax and cap-and-trade. However, we would like to emphasize that the two equivalences arise from different fundamental mechanisms.

To neutralize emissions in a carbon market, it must be that the price of pollution equals the cost of emission abatement in equilibrium. Since both tradable emission permits and carbon taxes can be considered as the price of pollution, they then must both equal to the emission abatement marginal cost. Hence, the carbon market literature concludes that the market-clearing price of per-ton carbon emission is the same as the carbon tax rate (e.g., Anand and Giraud-Carrier 2020, Requate 2006). However, in our paper, there does not exist such an "abatement marginal cost" to connect the market-clearing price and the marginal rate. Instead, the equivalence in our paper relies on introducing a new contract instrument—the allowance—and adjusting the contract price. Moreover, price discrimination does not exist in carbon taxing.

Not only does the equivalence of the sharing contract and the nonlinear contract hold from the perspective of the provider, but it also extends to subscribers. We present the result as follows.

Theorem 2 **(Equivalence to Subscribers)**. *Under the optimal sharing and nonlinear contracts, subscribers of the same type*

(i) consume the same amount of goods, i.e., $d_s(\theta) = d_n(\theta)$ *for* $\theta \geq \bar{\theta}_s = \bar{\theta}_n$*;*

(ii) receive the same surplus, i.e., $s_s(d_s(\theta) \mid \theta) = s_n(d_n(\theta) \mid \theta)$ *for* $\theta \geq \bar{\theta}_s = \bar{\theta}_n$*.*

The equivalence to subscribers also results from the allowance choice principle in (15) and the corresponding base price adjustment rule in Proposition 5(ii). Both collectively ensure that subscribers of the same type consume the same amount of goods and, moreover, pay the same effective costs under the respective contracts. As a result, for a type-θ subscriber,

$$s_s(d_s \mid \theta) = \theta d_s - (d_s)^2/2 - p_s - \hat{p}_s(d_s - Q_s)$$

$$
\begin{aligned}
&= \theta d_n - (d_n)^2/2 - p_n - \hat{p}_n(Q_s - Q_n) - \hat{p}_s(d_n - Q_s) \\
&= \theta d_n - (d_n)^2/2 - p_n - \hat{p}_n(d_n - Q_n) \\
&= s_n(d_n \mid \theta),
\end{aligned}
$$

where the second equality is due to $d_s = d_n$ and $p_s = p_n + \hat{p}_n(Q_s - Q_n)$ and the third equality results from $\hat{p}_s = \hat{p}_n$. And last, since customers of the same type receive the same individual surplus under both contracts, the two contracts must also result in the same market coverage.

Our equivalence result provides convincing evidence that the seemingly non-discriminatory sharing contact is indeed a form of price discrimination—high-consumption customers pay lower average unit prices. However, we also acknowledge that the performance equivalence to nonlinear pricing only holds when reselling is frictionless. As shown in Proposition 1, the service provider has no intention of inducing market frictions in the sharing process. Yet, there might be exogenous frictions, such as hassle costs associated with reselling. In this case, sharing pricing would underperform nonlinear pricing. Nevertheless, such exogenous frictions tend to be negligible for digital goods when resale could be achieved by several taps on customers' mobile devices.

Despite the theoretical equivalence, sharing pricing and nonlinear pricing have their own edges in practice. Conventional nonlinear pricing schemes, such as two- and three-part tariffs, typically rely on the marginal rate to differentiate customers based on their demands. This approach is appropriate when a customer's actual consumption can be accurately measured, such as mobile data usage. Thus, nonlinear pricing may be advantageous to sharing pricing due to the additional infrastructure cost of a trading platform. This is consistent with the CMHK's fade-away of its user trading platform. However, it can be challenging for many digital goods to have a consensus on the user's consumption. Consider cloud file hosting services, where individual storage space usage may vary significantly as users add or remove files. In such cases, adopting a two-part tariff may lead to ambiguity in defining actual usage—whether it's the maximum used space within a given time period, the average used space, or the space used at a specific time each month. This ambiguity arises because customer usage in cloud storage services may increase or decrease in a defined period of time, whereas the usage of other digital goods such as mobile data only increases. In light of this, sharing pricing emerges as an effective approach to profit from heterogeneous customers, particularly when their consumption levels may fluctuate either upward or downward in the measuring time frame, such as cloud storage services. This provides a theoretical explanation of Livedrive's long-lasting commitment to its resell program since 2010.

5.2. Generalization of the Equivalence

The key insight thus far is that reselling can function as a form of price discrimination when allowance is utilized as a contract instrument. The effectiveness of this approach to price discrimination can be comparable to nonlinear pricing due to the direct correspondence between the allowance and the marginal price of a nonlinear contract. We next reassure this insight by exploring two extensions of the base model in Section 4. Specifically, Section 5.2.1 confirms the equivalence for menu of contracts, while Section 5.2.2 demonstrates that the equivalence holds for uncertain demand.

5.2.1. Menu of Contracts. The equivalence of sharing pricing and nonlinear pricing extends to menu of contracts. Consider the same consumer population as specified in Section 4.1. Instead of offering a single-iter contract, the service provider offers a K-tier menu[10] of contracts for subscription. Specifically, let $\{p_{n_k}, Q_{n_k}, \hat{p}_{n_k}\}$, $k = 1, 2 \ldots, K$, represent a K-tier menu of nonlinear contracts, where subscribers of tier k pay p_{n_k} for Q_{n_k} units of allowance and $\hat{p}_{n_k}$ for each unit exceeding the allowance. Similarly, denote $\{p_{s_k}, Q_{s_k}\}$, $k = 1, 2 \ldots, K$, as a K-tier menu of sharing contracts, where subscribers of tier k pay p_{s_k} for Q_{s_k} units of allowance.

One particular consideration of the sharing menu is to specify the scope that peer-to-peer trading is allowed. The most restrictive scheme may only allow sharing among subscribers of the same tier and prohibit inter-tier allowance trading. On the other extreme, the most liberal scheme may have subscribers of all tiers to trade in the same resale market. The next lemma shows that the restrictive scheme yields a higher revenue for the service provider.

Lemma 5. *For a K-tier menu of sharing contracts, restricting trading to subscribers of the same tier generates a higher revenue at optimality than allowing all tiers to trade in the same resale market.*

If all tiers of the sharing contract menu are allowed to exchange in the same resale market, all subscribers who need to purchase additional usage allowance pay the same market-clearing price regardless of their types. Thus, customers have no interest in tiers with large allowances. As a result, all subscribers purchase the same tier from the menu, which hinders the provider's power in price discrimination. Consequently, restricting trading to subscribers of the same tier is preferable to the service provider. Moreover, the next result shows that restricting tradings within the same tier of sharing contract menu is as effective as the K-tier nonlinear menu.

Theorem 3 **(Equivalence of Menus)**. *If resales are restricted to subscribers of the same tier, the optimal K-tier sharing menu yields the same outcome as the optimal K-tier nonlinear menu.*

[10] The menu size K is assumed to be exogenous. Although the provider's optimal revenue grows in K, so does the menu administration cost. Thus, service providers usually offer small-sized menus in practice. Literature, e.g., Wilson (1993) and Bagh and Bhargava (2013), also suggests that a small-sized menu performs closely to the optimal menu.

Theorem 3 asserts that same-sized menus of the sharing contracts and nonlinear contracts are equivalent: the provider earns the same revenue and customers of the same type consume the same amount of goods for the same surplus. The pivotal building block of this equivalence originates from Proposition 5(ii), which suggests a direct approach to constructing an outcome-equivalent tier between the optimal menus of the sharing contracts and nonlinear contracts.

5.2.2. The Impact of Uncertainty on the Equivalence. We next examine the equivalence of the sharing contract and the nonlinear contract under demand uncertainty, which may arise from idiosyncratic valuation shocks.

Consider a group of mobile data subscribers who work as business consultants and value each unit of mobile data at θ. Depending on how many hours they spend in the office and on travel, the realized valuations θ's may differ: availability of alternatives for Internet access in office may depreciate the value of mobile data, whereas the same unit of data may become more valuable as the number of hours spent on travel increases. To model these idiosyncratic shocks, we consider an additive type-specific perturbationon the intrinsic valuation θ. Specifically, for each type-θ customer, his realized valuation of unit usage equals $\theta + \epsilon_\theta$, where ϵ_θ is a valuation perturbation drawn from a distribution $G_\theta(\cdot)$ with zero mean, i.e., $\mathsf{E}[\epsilon_\theta] = 0$, on the support of $[-\epsilon_\theta^l, \epsilon_\theta^u]$ where $\epsilon_\theta^l > 0$ and $\epsilon_\theta^u > 0$. Our uncertainty model is fairly general. Customers of various types can have distinct valuation perturbation distributions $G_\theta(\cdot)$'s and these $G_\theta(\cdot)$'s are not required to have particular structure properties.

The idiosyncratic shocks to valuations lead to uncertain individual demands. To illustrate, let us consider customers' consumption decisions under the sharing contract. Assume that a type-θ customer subscribes to the service but experiences a perturbation ϵ_θ. Thus, her realized surplus is

$$s_s(d_s \mid \theta + \epsilon_\theta) = u(d_s \mid \theta + \epsilon_\theta) - c_s(d_s \mid \theta + \epsilon_\theta) = (\theta + \epsilon_\theta) d_s - \frac{1}{2} d_s^2 - (p_s + \hat{p}_s \cdot (d_s - Q_s)). \tag{19}$$

It is straightforward to see that the surplus-maximizing demand satisfies

$$d_s(\theta + \epsilon_\theta) = \max\{\theta + \epsilon_\theta - \hat{p}_s, 0\}. \tag{20}$$

Since ϵ_θ is a random draw from $G_\theta(\cdot)$, the optimal demand $d_s(\theta + \epsilon_\theta)$ is thus ex ante uncertain.

Consequently, whether customers subscribe to a sharing contract is determined by their expected surplus under uncertainty. Specifically, type-θ customers subscribe if and only if

$$\begin{aligned}
\overline{s}_s(p_s, Q_s \mid \theta) &= \mathsf{E}_{\epsilon_\theta}\left[s_s(d_s \mid \theta + \epsilon_\theta)\right] \\
&\overset{(20)}{=} \mathsf{P}(\theta + \epsilon_\theta \geq \hat{p}_s)\mathsf{E}_{\epsilon_\theta}\left[s_s(d_s) \mid \theta + \epsilon_\theta \geq \hat{p}_s\right] + \mathsf{P}(\theta + \epsilon_\theta < \hat{p}_s)\mathsf{E}_{\epsilon_\theta}\left[s_s(d_s) \mid \theta + \epsilon_\theta < \hat{p}_s\right] \\
&= \mathsf{P}(\theta + \epsilon_\theta \geq \hat{p}_s)\mathsf{E}_{\epsilon_\theta}\left[(\theta + \epsilon_\theta - \hat{p}_s)^2/2 - p_s + \hat{p}_s Q_s \mid \theta + \epsilon_\theta \geq \hat{p}_s\right] + \\
&\quad\ \mathsf{P}(\theta + \epsilon_\theta < \hat{p}_s)\mathsf{E}_{\epsilon_\theta}\left[-p_s + \hat{p}_s Q_s \mid \theta + \epsilon_\theta < \hat{p}_s\right] \\
&= \mathsf{P}(\theta + \epsilon_\theta \geq \hat{p}_s)\mathsf{E}_{\epsilon_\theta}\left[(\theta + \epsilon_\theta - \hat{p}_s)^2/2 \mid \theta + \epsilon_\theta \geq \hat{p}_s\right] - p_s + \hat{p}_s Q_s
\end{aligned} \tag{21}$$

$$\geq \quad 0.$$

We note that the customer's expected surplus is not monotone in his (intrinsic) type θ because of the general uncertainty framework. Specifically, it is not always true that if $\theta > \theta$ then $\overline{s}_s(p_s, Q_s \mid \theta) > \overline{s}_s(p_s, Q_s \mid \theta\ )$ or vice versa. Therefore, customers' subscription decisions do not necessarily follow a threshold policy as in the case without uncertainty (see Lemma 1). For exposition, we introduce

$$\Theta_s(p_s, Q_s) := \{\theta \mid \overline{s}_s(p_s, Q_s \mid \theta) \geq 0\} \text{ and } \mu(\Theta_s(p_s, Q_s))$$

to denote the subscriber set and its probability measure under the sharing contract with uncertainty.

The provider's revenue problem is to set the base price p_s and the allowance Q_s so that

$$\max_{p_s, Q_s} \overline{\Pi}_s(p_s, Q_s) = p_s \cdot \mu(\Theta_s(p_s, Q_s)) \quad \text{s.t. } p_s \geq 0 \text{ and } Q_s \geq 0.$$

In TS3, we derive corresponding customer demands, individual surplus, and the provider's revenue for the bucket and nonlinear contracts.

In the presence of uncertainty, the equivalence of the sharing contract and the nonlinear contract remains to a lesser extent. We acknowledge that the optimal sharing contact yields the same revenue as the optimal two-part tariff—a special type of nonlinear contract with zero allowance; i.e., $Q_n = 0$. In more general cases, we prove that when uncertainty is present, a nonlinear contract, in particular a three-part tariff, yields no less revenue at optimality than a sharing contract, which still outperforms a bucket contract.

Proposition 6 **(Equivalence to Two-Part Tariffs under Uncertainty)**. *Let* $\overline{\Pi}_s$ *and* $\overline{\Pi}_n^0$ *be the optimal revenues of the sharing contract and the two-part tariff, i.e., the nonlinear contract with zero allowance, with uncertain valuations. Moreover, denote the corresponding subscriber sets as* Θ_s *and* Θ_n^0 *. Then, we have (i)* $\overline{\Pi}_s = \overline{\Pi}_n^0$ *and (ii)* $\Theta_s = \Theta_n^0$ *.*

The equivalence of sharing contracts and two-part tariffs in Proposition 6 resembles a similar intuition in the deterministic case in Section 5.1. The service provider chooses the allowance to maneuver the market clearing price indirectly to be aligned with the overage rate of the two-part tariff. Such a market clearing price gives rise to equal consumption by customers of the same types under both contracts for the same magnitude of uncertainty. By charging the base price of the two-part tariff plus the surcharge for the allowance at a unit price that equals the market clearing price, the provider effectively collects the same revenue from the sharing contract.

The equivalence of the sharing contract and the two-part tariff is preserved due to their neutrality toward uncertainty. Under two-part tariffs, subscribers pay for what they consume. Their effective costs are higher when they are undergoing upward shocks and lower when they are undergoing downward ones. Thus, the uncertainty is expected to affect the provider's revenue equally in both positive and negative ways. The sharing contract preserves this neutrality. Although the fixed allowance seems to hinder subscribers from consuming beyond the allowance when there are upward shocks or from saving some costs when there are downward ones, the peer-to-peer sharing helps neutralize these variations so that the provider is equally affected by shocks in both directions on expectation. As a result, the sharing contract and two-part tariff perform identically at optimality.

Nonetheless, the equivalence breaks down when the nonlinear contract has a nonzero allowance.

Proposition 7 **(Dominance of Nonlinear Contracts under Uncertainty)**. *Let* $\overline{\Pi}_s$ *and* $\overline{\Pi}_n$ *denote the optimal revenues of the sharing and nonlinear contracts with uncertain valuations, respectively. Then, we have* $\overline{\Pi}_s \leq \overline{\Pi}_n$.

Proposition 7 shows that the sharing contract underperforms the nonlinear contract, in particular the three-part tariff, in the presence of uncertainty. The discrepancy results from the uneven effects of upward and downward uncertainties on a three-part tariff. To illustrate, let us compare a three-part tariff with a two-part one. Without an allowance, the provider who employs a two-part tariff earns more revenues only when subscribers undergo upward shocks and consume more. In other cases where subscribers have downward shocks and use less, the two-part tariff in fact works in favor of subscribers by charging them only for what they consume, which results in less revenue for the provider. In contrast, a three-part tariff can exploit uncertainties of both directions. Intuitively, in the presence of uncertainty, customers would like to pay for some "safety allowance" to hedge against demand shocks. The "safety allowance" certainly costs subscribers more and only justifies its value when subscribers consume more than their expectations. If the realized uncertainty is a downward shock that reduces a subscriber's consumption, the "safety allowance" is useless but is already paid for in the base price. We use Example 1 to make this argument more concrete.

Example 1. Assume that customer types follow a uniform distribution on $[0, 1]$ and moreover that shocks happening to type-θ customers are random draws from $[-\theta, \theta]$ with equal probability. The optimal two-part tariff has the form of $(p_n^0, \hat{p}_n^0) = (0.1419, 0.2171)$ with a market coverage $1 - \bar{\theta}_n^0 = 1 - 0.6168 = 0.3832$ and an optimal revenue $\Pi_s^0 = 0.1048$. The optimal three-part tariff is contracted as $(p_n, Q_n, \hat{p}_n) = (0.1854, 0.3129, 0.2561)$ with a market coverage $1 - \bar{\theta}_n = 1 - 0.6127 = 0.3873$ and an optimal revenue $\Pi_n = 0.1055$. Under the optimal three-part tariff, the total allowance offered equals $(1 - \bar{\theta}_n)Q_n = 0.12119$, out of which 0.0119 of a paid unit is not expected to be used. The provider thus earns more revenue from the three-part tariff.

Example 1 illustrates the impact of uncertainty when comparing a three-part tariff with a two-part one. Our Proposition 4 and Bhargava and Gangwar (2018) (see Corollary 2 on p. 1523) both imply that if customer heterogeneity has an IFR an optimal three-part tariff yields the same revenue as an optimal two-part tariff in the absence of uncertainty. Example 1, however, provides an example in which an IFR does not preserve the equivalence under uncertainty due to the asymmetric impacts of uncertainty on revenue.

6. Overage and Underage Disutility

We have so far considered risk-neutral customers. In this section, we explore how consumer psychological costs may affect the effectiveness of sharing contracts and nonlinear contracts. Specifically, we consider consumers' psychological response to overage or underage beyond the allowance: a customer incurs a disutility when her consumption d either exceeds or falls short of her allowance Q. We define the utility received by a type-θ customer as

$$u(d\mid\theta)=\theta d-\frac{1}{2}d^2-\frac{1}{2}w_o[(d-Q)^+]^2-\frac{1}{2}w_u[(Q-d)^+]^2, \tag{22}$$

where $w_o\geq 0$ and $w_u\geq 0$ are the unit overage and underage costs, respectively.

When being offered a sharing contract, a type-θ customer's surplus can be expressed as

$$s_s(d_s\mid\theta)=\theta d_s-\frac{1}{2}d_s^2-\frac{1}{2}w_o[(d_s-Q_s)^+]^2-\frac{1}{2}w_u[(Q_s-d_s)^+]^2-p_s-\hat{p}_s(d_s-Q_s) \tag{23}$$

and she solves $\max_{d_s\geq 0}\ s_s(d_s\mid\theta)$ for the optimal demand $d_s(\theta)$ and subscribes if $s_s(d_s\mid\theta)\geq 0$.

Lemma TS6 characterizes the demand of a type-θ customer. Lemma TS7 and Proposition TS1 further develop the equilibrium of the sharing market for a given sharing contract (p_s,Q_s). Optimizing over (p_s,Q_s), we derive the optimal sharing contract term below.

Proposition 8 **(Optimal Sharing Contract)**. *It is optimal to offer a sharing contract so that there are no speculators subscribing in equilibrium. Under such an optimal sharing contract,*

(i) the provider offers a contract (p_s,Q_s) with the resulting equilibrium market clearing price $\hat{p}_s$, where p_s, Q_s, and $\hat{p}_s$ are the solution to

$$\begin{cases}\Big(\int_{Q_s}^{\hat{p}_s+Q_s}\frac{1}{1+w_u}f(\theta)\,d\theta+\int_{\hat{p}_s+Q_s}^{\Theta}\frac{1}{1+w_o}f(\theta)\,d\theta\Big)\Big(\frac{\bar{F}(Q_s)}{f(Q_s)}(\hat{p}_s-(1+w_u)Q_s)+\frac{\hat{p}_s^2+(1+w_u)Q_s^2}{2}\Big)=\frac{\hat{p}_s^2\bar{F}(Q_s)}{1+w_u},\\ Q_s\bar{F}(Q_s)=\int_{Q_s}^{\hat{p}_s+Q_s}\frac{\theta-\hat{p}_s+w_uQ_s}{1+w_u}f(\theta)\,d\theta+\int_{\hat{p}_s+Q_s}^{\Theta}\frac{\theta-\hat{p}_s+w_oQ_s}{1+w_o}f(\theta)\,d\theta,\\ \hat{p}_s=\sqrt{(1+w_u)(2p_s-Q_s^2)}.\end{cases} \tag{24}$$

(ii) customers subscribe to the service if and only if $\theta\geq\bar{\theta}_s=Q_s$.

Proposition 8 generalizes Proposition 3, which can be considered as a special case of $w_o=w_u=0$.

Similarly, when being offered a nonlinear contract $(p_n,Q_n,\hat{p}_n)$, a type-θ subscriber solves the following problem

$$\max_{d_n\geq 0}\ s_n(d_n\mid\theta)=\theta d_n-\frac{1}{2}d_n^2-\frac{1}{2}w_o[(d_n-Q_n)^+]^2-\frac{1}{2}w_u[(Q_n-d_n)^+]^2-p_n-\hat{p}_n(d_n-Q_n)^+ \tag{25}$$

for her optimal demand. Lemmas TS8 and TS9 characterize the optimal consumption level and identifies the subscribers, respectively. Solving the revenue-maximization problem in (12). We obtain the following optimal contract term.

PROPOSITION 9 **(Optimal Nonlinear Contract)**. *Suppose that Assumption 1 holds. Under the optimal nonlinear contract,*

(i) customers subscribe to the service if and only if her type $\theta \geq \bar{\theta}_n$*, where* $\bar{\theta}_n$ *is the solution to*

$$\frac{(1+w_o)^2\bar{\theta}_n^2 - \left(\frac{\int_{\bar{\theta}_n}^{\Theta}\theta f(\theta)\,d\theta}{\bar{F}(\bar{\theta}_n)} - \bar{\theta}_n\right)^2}{2\left[(1+w_o)^2\bar{\theta}_n - \frac{\int_{\bar{\theta}_n}^{\Theta}\theta f(\theta)\,d\theta}{\bar{F}(\bar{\theta}_n)} + \bar{\theta}_n\right]} = \frac{\bar{F}(\bar{\theta}_n)}{f(\bar{\theta}_n)};$$

(ii) the optimal price p_n *and optimal allowance* Q_n *are*

$$p_n = \frac{(\bar{\theta}_n - \hat{p}_n)^2}{2} + \hat{p}_n(\bar{\theta}_n - \hat{p}_n),\ Q_n = \bar{\theta}_n - \hat{p}_n,$$

where $\hat{p}_n = \left(\int_{\bar{\theta}_n}^{\Theta}\theta f(\theta)\,d\theta/\bar{F}(\bar{\theta}_n) - \bar{\theta}_n\right)/(1+w_o)$ *is the optimal overage rate.*

The equivalence of the sharing contact and the nonlinear contract does not hold when subscribers incur underage or overage disutility. Either may dominate depending on the unit overage and underage costs.

THEOREM 4 **(Comparison of Revenue)**. *For any given unit overage cost* w_o*, there always exists a threshold* $\gamma > 1$ *such the sharing (nonlinear) contract yields weakly higher revenue than the nonlinear (sharing) contract if* $w_o \geq (\leq)\gamma w_u$*.*[11]

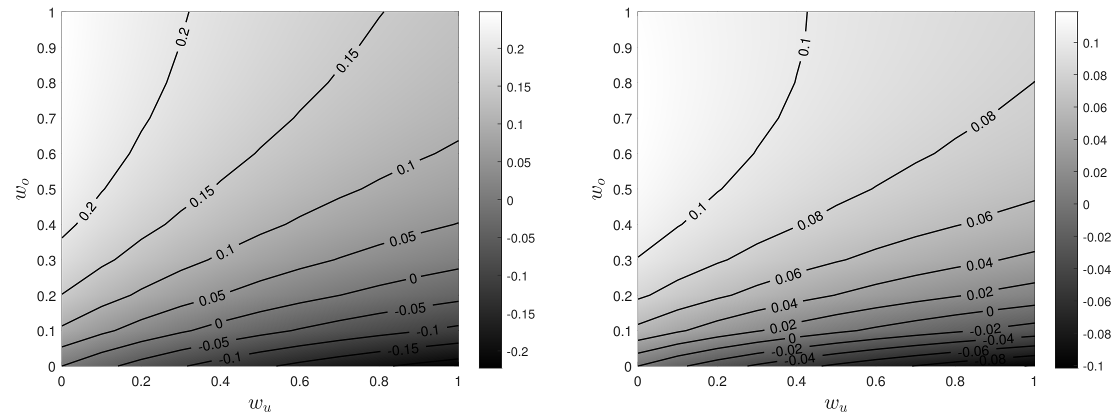
Theorem 4 compares sharing and nonlinear contracts. We illustrate the result in Figure 2 using the uniform distribution in $[0, 10]$ and exponential distribution with mean $1/\lambda = 2$.[12] Figure 2 displays the discrepancy of the optimal revenues of the sharing and nonlinear contracts. The discrepancy $\Pi_s - \Pi_n$ tends to decrease in the underage rate w_u but increase in the overage rate w_o.

Note that Proposition 9 implies that the optimal nonlinear contract sets the allowance Q_n low so that *all* subscribers have to pay at the marginal rate $\hat{p}_n$ to consume more than the allowance. Hence, subscribers of a nonlinear contact do not incur underage disutility, whereas underage disutility always occurs to some subscribers of the sharing contract and impedes the revenue the service provider can extract. Apparently, the larger the unit underage cost w_u is, the more severe the underage disutility hurts the provider's revenue yielded from a sharing contract. Thus, $\Pi_s - \Pi_n$ decreases in w_u.

[11] The equality is only achieved at $w_o = \gamma w_u$.

[12] Lemma TS10 reveals that the revenue comparisons under uniform and exponential distributions are independent on the distribution parameters. Thus, observations in Figure 2 also apply to any uniform or exponential distribution.

Figure 2 Comparison of Revenue ($\Pi_s - \Pi_n$)



(a) Uniform Distribution in $[0, 10]$ (b) Exponential Distribution with $\lambda = 0.5$

The other impact of a relatively small allowance Q_n of a nonlinear contract is that every subscriber has to incur the overage disutility. As the unit overage rate w_o increases, subscribers reduce their demands and hurt the provider's revenue yielded from a nonlinear contract. In this case, a sharing contract can be an effective alternative: the optimal sharing contract sets the allowance $Q_s > Q_n$ so that some subscribers of the sharing contract can avoid the overage cost. Thus, the higher the unit overage cost w_o is, the more effective the sharing contract is. Thus, $\Pi_s - \Pi_n$ increases in w_o.

The monotonicity of $\Pi_s - \Pi_n$ in w_o and w_u implies that there exists some constant γ so that the sharing contract dominates if and only if $w_o \geq \gamma w_u$ and vice versa.

7. Concluding Remarks

In this paper, we employ a game-theoretic model to examine how the distinctive features of digital goods, particularly their traceability and control over usage allowance, can be leveraged to address the cannibalization problem caused by peer-to-peer resales.

We compare the sharing pricing, under which subscribers of a specific usage allowance can trade unused allowance, with the nonlinear pricing, e.g., a two-part tariff, which is known to be effective in managing heterogeneous demands. We show that this non-discriminatory sharing contract which promotes reselling is equivalent to implementing a price discrimination strategy via a two-part tariff. Not only do the two contracts yield identical revenue, but they also achieve the same market coverage and result in the same demand and individual surplus for customers of the same type. This equivalence holds for menus of contracts or uncertain consumer demand.

The optimal contract terms of the sharing pricing implies that the equivalence to the nonlinear pricing, e.g., the two-part tariff, has nothing to do with the traceability of digital goods resales in the secondary market. Even if the seller could take a cut from all resales, she chooses not to do so at optimality. Instead, the seller capitalizes on controlling both the usage allowance and the price to maximize revenue when the sharing contract is initially sold. By carefully selecting an allowance, peer-to-peer resales under the sharing pricing reallocate uniformly distributed allowances in the contract to customers with various demands, effectively practicing price discrimination at a market-clearing price identical to the marginal price under nonlinear pricing. Hence, sharing pricing for digital goods serves as an equivalent alternative to price discrimination. When nonlinear pricing is not feasible, sharing pricing can offer a valuable alternative pricing approach.

Our paper underscores how conventional pricing theory may not be applicable to digital goods. The unique characteristics of digital goods may introduce numerous novel pricing schemes beyond traditional bucket pricing and provide new tools unavailable for physical goods.

Finally, it is essential to validate our theoretical framework through complementary empirical research in the future. Transaction-level data, encompassing daily trading volume and fluctuations in equilibrium trading prices over time, not only provides tangible evidence to validate the theoretical constructs in this study but also offers insights to evaluate the significance of the trading market.

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EC1. Bucket Contract

We present our results of the bucket contract in this section. In particular, Section EC1.1 characterizes the optimal term of the bucket contract, Section EC1.2 and Section EC1.3 compares its performance with the sharing and nonlinear contracts with no uncertainty and under uncertainty, respectively.

EC1.1. Bucket Pricing

We consider the bucket contract in which the provider charges each subscriber p for consumption up to Q units.[13] Therefore, there is no discrimination among customers. For example, Dropbox, a file hosting service provider, uses this pricing scheme and offers 2TB cloud storage space at \$9.99 a month. In this simple bucket contract, usage beyond the Q-unit limit is prohibited. Hence, $c(d \mid \theta) = p$ where $d \leq Q$ for all subscribers. A type-θ customer solves the following problem

$$\max_{0 \leq d \leq Q} \; s(d \mid \theta) = u(d \mid \theta) - c(d \mid \theta) = \theta d - \frac{1}{2} d^2 - p \tag{EC.1}$$

to determine whether to subscribe and the consumption level $d(\theta)$ if subscribing. The next lemma characterizes these customers' decisions for a given bucket contract (p, Q).

Lemma EC1. *Assume that the service provider offers a bucket contract (p, Q). Then,*

(i) If $0 \leq p < Q^2/2$, customers subscribe if and only if $\theta \geq \bar{\theta} = \sqrt{2p}$ and consume $d\ (\theta) = \min\{\theta, Q\}$ units of the goods;

(ii) If $Q^2/2 \leq p \leq Q^2/2 + \Theta Q$, customers subscribe if and only if $\theta \geq \bar{\theta} = p/Q + Q/2$ and consume $d\ (\theta) = Q$ units of the goods;

(iii) If $p > Q^2/2 + \Theta Q$, no customers subscribe.

Lemma EC1 first confirms an intuitive result: customers subscribe to the contract only when the price is low relative to the allowance offered, i.e., cases (i) and (ii). Second, Lemma EC1 reveals that customers' subscription decisions follow a simple threshold structure: only those who value each unit of the goods more than $\bar{\theta}$ subscribe to the service. Third, Lemma EC1 shows that the simple non-discriminatory bucket contract is not able to satisfy customers' heterogeneous demands. In case (i), there will be some subscribers who would not consume all Q units and leave some unused goods. In contrast, in both cases (i) and (ii), there are always some subscribers who have to downgrade their usage to Q units even though they would like to consume more.

Cases (i) and (ii) in Lemma EC1 imply two potential pricing tactics for the service provider at a given allowance. On the one hand, the provider may offer an affordable contract as in case (i) to achieve a high market coverage with some subscribers consuming less than the allowance.

[13] We use "contract" and "pricing" to refer to the terms of a service agreement and the process of determining the terms, respectively.

Alternatively, the provider could also charge a higher premium as in case (ii) so that all subscribers use up all their allowances. We next characterize the optimal contract $(p\ , Q\ )$.

Since customer subscription behavior follows a threshold structure, we formulate the provider's revenue-maximization problem as

$$\max_{p,Q} \ \Pi(p,Q) = p \cdot \bar{F}(\bar{\theta}(p,Q)) \quad \text{s.t. } p \geq 0 \text{ and } Q \geq 0 \tag{EC.2}$$

where $\bar{F}(\bar{\theta}(p,Q))$ represents the fraction of customers who subscribe to the service.

Proposition EC1 **(Optimal Bucket Contract)**. *Under the optimal bucket contract,*

(i) the provider charges p *for the service, where* p *is the solution to*

$$\frac{\bar{F}(\sqrt{2p})}{\sqrt{2p}\, f(\sqrt{2p})} = \frac{1}{2}, \tag{EC.3}$$

and offers at least $Q = \sqrt{2p}$ *units of allowance;*

(ii) customers subscribe if and only if $\theta \geq \bar{\theta} = \sqrt{2p}$ *and their demand* $d\ (\theta) = \min\{\theta, Q\ \}$.

Proposition EC1 first shows that the optimal bucket contract has a unique price defined by (EC.3). However, any allowance exceeding $\sqrt{2p}$ units yields the same revenue. At the price of p with an allowance of $Q = \sqrt{2p}$, all subscribers earn strictly positive net surplus except type-$\bar{\theta}$ customers whose net surplus is zero and consumption level $d(\bar{\theta}\ )$ equals to $\sqrt{2p}$. Maintaining the price at p but increasing the allowance Q beyond $\sqrt{2p}$ would not induce type-$\bar{\theta}$ customers to consume more as their demands are already fully satisfied by $Q = \sqrt{2p}$ units of allowance. Therefore, type-$\bar{\theta}$ customers still earn zero net surpluses with more than $Q = \sqrt{2p}$ units of allowance at price p and so do all non-subscribers. Consequently, lifting the optimal allowance beyond $\sqrt{2p}$ would not increase the market coverage and the provider's revenue. It, however, allows subscribers of type $\theta > \bar{\theta}$ to consume more, some of whom will also have an unused allowance, and thus improves the net surplus of these subscribers. Moreover, among all optimal bucket contracts identified in Proposition EC1, cases (i) and (ii) of Lemma EC1 can both occur depending on the choice of the allowance Q at p .

EC1.2. Superiority of Sharing and Nonlinear Contracts Over Bucket Contracts

We now establish the superiority of the sharing and nonlinear contracts over the bucket contract and discuss the pros and cons when each is implemented in practice.

All three kinds of contracts include two common parameters, the base price, and the corresponding allowance. While the bucket contract relies only on these two parameters, the other two kinds of contracts have other instruments: a peer-to-peer sharing market under the sharing contract and an overage rate under the nonlinear contract. The next proposition shows that the additional instruments benefit not only the service provider but also the customers.

Proposition EC2 **(Comparisons under Fixed Base Price and Allowance)**. *Consider a bucket contract* (p,Q), *a sharing contract* (p_s,Q_s), *and a nonlinear contract* $(p_n,Q_n,\hat{p}_n \geq 0)$ *such that* $p = p_s = p_n$, $Q = Q_s = Q_n$. *Assume that there are nonzero subscribers under all contracts. Then, we have* *(i)* *the sharing and nonlinear contracts induce no less market coverage than the bucket contract, i.e.,* $\bar{\theta}_s \le \bar{\theta}$ *and* $\bar{\theta}_n \le \bar{\theta}$*;*

(ii) *the sharing and nonlinear contracts lead to no less individual surplus than the bucket contract, i.e.,* $s_s(d_s(\theta) \,|\, \theta) \ge s(d\ (\theta) \,|\, \theta)$ *and* $s_n(d_n(\theta) \,|\, \theta) \ge s(d\ (\theta) \,|\, \theta)$ *for any given* $\theta \quad [0, \Theta]$.

(iii) *the sharing and nonlinear contracts yield no less revenue than the bucket contract, i.e.,* $\Pi_s(p_s, Q_s) \ge \Pi(p, Q)$ *and* $\Pi_n(p_n, Q_n, \hat{p}_n) \ge \Pi(p, Q)$.

Proposition EC2 compares all three contracts by fixing their common parameters—the price and the allowance. The inferiority of a bucket contract results from its inability to discriminate among customer heterogeneous demands due to its uniformity. In contrast, nonlinear contracts implement price discrimination by charging an overage rate. Thus customers can buy more to meet their heterogeneous needs, improve their surplus, and also do so at a lower average unit price. As a result, service subscriptions grow and the provider's revenue increases. As noted in the literature (e.g., Oi 1971, Tirole 1988, Wilson 1993), nonlinear contracts, e.g., two-part tariffs, render arbitrage opportunities and thus their practical application has to prevent resale. Otherwise, the service provider may lose revenue when certain customers buy more than they would use at low prices and then resell to others at high prices. The bucket contract, on the other hand, eliminates the arbitrage possibility by imposing a uniform unit price for all goods sold.

The sharing contract not only inherits the bucket contract's immunity from arbitrage resale but also meets customers' heterogeneous demands more effectively via peer-to-peer sharing. On the one hand, the sharing process allows high-demand customers to buy more of the goods from low-demand customers. The allowance exchange re-allocates evenly distributed goods to meet customer heterogeneous demands, leading to a higher individual surplus. On the other hand, the possibility of selling unused allowance essentially reduces the entry barrier of the service and makes it more affordable to low-demand customers. Consequently, more customers subscribe and that increases the provider's revenue in comparison with the bucket contract.

Proposition EC2 shows the advantages of the sharing and nonlinear contracts over the bucket contract when both have the same price and allowance. The next proposition further compares them when each one is written with the optimal terms.

Proposition EC3 **(Comparisons under Optimal Terms)**. *Consider the optimal bucket contract* $(p\ , Q\ )$*, the optimal sharing contract* (p_s, Q_s)*, and the optimal nonlinear contract* $(p_n, Q_n, \hat{p}_n)$*. Then, we have*

(i) *the optimal sharing and nonlinear contracts induce no less market coverage than the optimal bucket contract, i.e.,* $\bar{\theta}_s \le \bar{\theta}$ *and* $\bar{\theta}_n \le \bar{\theta}$ *;*

(ii) *the optimal sharing and nonlinear contracts yield no less revenue than the optimal bucket contract, i.e.,* $\Pi_s(p_s, Q_s) \ge \Pi(p\ , Q\ )$ *and* $\Pi_n(p_n, Q_n, \hat{p}_n) \ge \Pi(p\ , Q\ )$.

(iii) the optimal allowances under the sharing and nonlinear contracts are no more than that under the optimal bucket contract, i.e., $Q_s \leq Q$ *and* $Q_n \leq Q$ *, but the total consumption at optimality is no less than that under the optimal bucket contract, i.e.,* $\int_{\bar{\theta}_s}^{\Theta} d_s(\theta)dF(\theta) \geq \int_{\bar{\theta}}^{\Theta} d\ (\theta)dF(\theta)$ *and* $\int_{\bar{\theta}_n}^{\Theta} d_n(\theta)dF(\theta) \geq \int_{\bar{\theta}}^{\Theta} d\ (\theta)dF(\theta)$.

Given the result in Proposition EC2(iii), the sharing and nonlinear contracts can easily outperform the optimal bucket contract by charging the same price and setting the same allowance. Therefore, Proposition EC3(ii) follows readily.

As for the market coverage, Proposition EC3(i) confirms that the optimal sharing and nonlinear contracts have an advantage over the optimal bucket contract. That finding is, however, not so obvious, since market coverage under these optimal contracts is in general achieved at different prices and allowances. We attribute this finding to the efficiency of the sharing and nonlinear contracts in fulfilling heterogeneous demands. The uniformity of the bucket contract hinders the provider from price-discriminating these heterogeneous customers. To maximize her revenue, the provider designs the bucket contract to cater to high-valuation customers, since not only are they willing to pay more but they also consume more. Therefore, the optimal bucket contract tends to include a relatively large allowance at a high price, which reduces the affordability of the service and results in low market coverage. The sharing and nonlinear contracts, in contrast, each have their own instruments for catering to high-valuation customers with high demands. Therefore, they can offer lower allowances at affordable prices and attract more subscribers. Despite lower allowances, both contracts induce higher total consumption than the bucket contract thanks to their additional instruments for serving heterogeneous demands, coupled with a larger market coverage.

EC1.3. Bucket Contract under Uncertainty

Although bucket contract is not as effective as the nonlinear contract, e.g., a two-part tariff, it is ambiguous if this ineffectiveness still retains under demand uncertainty that is specified in Section 5.2.2. The advantage of a two-part tariff is its price-discrimination capability, whereas the disadvantage is its inability to extract revenues under downward uncertainty. In contrast, a bucket contract fails to distinguish heterogeneous customers but prospers from both upward and downward uncertainties through "safety allowance." Therefore, it is ambiguous whether two-part tariffs still perform better than bucket pricing under uncertainty. The next result helps us solve the mystery of bucket pricing and two-part tariffs under uncertainty.

Proposition EC4 **(Bucket vs. Sharing and Nonlinear Contracts under Uncertainty)**. *Let* $\overline{\Pi}\ $, $\overline{\Pi}_s$, *and* $\overline{\Pi}_n$ *denote the optimal revenues of the bucket, sharing, and nonlinear contracts with uncertain valuations, respectively. Then, we have* $\overline{\Pi}\ \leq \overline{\Pi}_s \leq \overline{\Pi}_n$.

Proposition EC4 gives a clear answer by using sharing pricing as a stepping stone. By Proposition EC4, sharing pricing outperforms bucket pricing in the presence of uncertainty. That is because sharing contracts can always offer the same terms as bucket contracts but with peer-to-peer trading opportunities, which would only improve the market coverage and profitability. Recalling the equivalence of sharing pricing and two-part tariffs in Proposition 6, Proposition EC4 thus implies that two-part tariffs also dominate bucket pricing even under uncertainty.

EC2. Proofs in Main Body

Proof of Proposition 2. For ease of exposition, we define $g(y) = \int_y^{\Theta} \theta f(\theta)\mathrm{d}\theta - y\bar{F}(y)$. Note that $g(y)$ strictly decreases in $y \geq 0$. Hence, $\max_{y\geq 0} g(y) = \mathsf{E}[\theta]$ at $y = 0$ and $\min_{y\geq 0} g(y) = 0$ at $y = \Theta$.

(i) *Necessity.* Suppose that the sharing market has an unique equilibrium with speculators. By Lemma 2(i), we have $0 \leq \bar{\theta}_s < \hat{p}_s$ and $\hat{p}_s \geq p_s/Q_s$. Consider two cases: $\bar{\theta}_s = 0$ and $\bar{\theta}_s > 0$.

If $\bar{\theta}_s = 0$, $Q_s = Q_s\bar{F}(\bar{\theta}_s) = \int_{\hat{p}_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta - \hat{p}_s\bar{F}(\hat{p}_s) \leq \int_{p_s/Q_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta - (p_s/Q_s)\bar{F}(p_s/Q_s) = \overline{Q}_s\bar{F}(p_s/Q_s) < \overline{Q}_s$, where the first inequality is due to $g(y)$ strictly decreases in $y \geq 0$ and $\hat{p}_s \geq p_s/Q_s$.

If $\bar{\theta}_s > 0$, we first prove $\hat{p}_s = p_s/Q_s$ by contradiction. Suppose $\hat{p}_s > p_s/Q_s$. For customers of type $\theta < \bar{\theta}_s$, if they subscribe, $d_s(\theta) = \max\{\theta - \hat{p}_s, 0\} = 0$ and $s_s(d_s(\theta) \mid \theta) = \hat{p}_s Q_s - p_s > 0$, which contradicts with the fact that only customers with $\theta \geq \bar{\theta}_s$ subscribe. Since $0 \leq \bar{\theta}_s < \hat{p}_s$, then $Q_s\bar{F}(\bar{\theta}_s) = \int_{\hat{p}_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta - \hat{p}_s\bar{F}(\hat{p}_s) > Q_s\bar{F}(\hat{p}_s)$ due to the monotonicity of $\bar{F}(\cdot)$. Note $\hat{p}_s = p_s/Q_s$, we have

$$\int_{\hat{p}_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta - \hat{p}_s\bar{F}(\hat{p}_s) > Q_s\bar{F}(\hat{p}_s) \qquad Q_s < \int_{p_s/Q_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta/\bar{F}(p_s/Q_s) - p_s/Q_s = \overline{Q}_s.$$

Sufficiency. Suppose $0 \leq Q_s < \overline{Q}_s$. To ensure the existence of a sharing equilibrium with speculators, we need to show that (5) has a unique solution $\hat{p}_s \geq 0$ and there exists a unique $0 \leq \bar{\theta}_s < \hat{p}_s$ such that $s_s(d_s(\bar{\theta}_s) \mid \bar{\theta}_s) \geq 0$, where the equality is achieved if $\bar{\theta}_s = 0$. Let us consider two cases: (a) $0 \leq Q_s < g(p_s/Q_s)$, and (b) $g(p_s/Q_s) \leq Q_s < \overline{Q}_s$.

(a) If $0 \leq Q_s \leq g(p_s/Q_s)$, we first prove $\bar{\theta}_s > 0$ does not occur in equilibrium. Then, we construct a sharing with speculators equilibrium with $\bar{\theta}_s = 0$ and show that this is the only possible equilibrium.

Suppose $\bar{\theta}_s > 0$ in equilibrium. From the proof of necessity, we know $\hat{p}_s = p_s/Q_s$ if $\bar{\theta}_s > 0$. Then,

$$Q_s\bar{F}(\bar{\theta}_s) < Q_s \leq g(p_s/Q_s) = g(\hat{p}_s),$$

which implies (5) has no solution. Hence, it is not possible to have $\bar{\theta}_s > 0$ in equilibrium.

Next let $\bar{\theta}_s = 0$ and we show that there exists a unique $\hat{p}_s > p_s/Q_s$ such that the market-clearing condition (5) holds. Note that $Q_s\bar{F}(\bar{\theta}_s = 0) = Q_s$. Hence, we can rewrite (5) as $Q_s = g(\hat{p}_s)$. Since $g(\hat{p}_s)$ $[0, \mathsf{E}[\theta]]$ for $\hat{p}_s \geq 0$, there must exists a $\hat{p}_s$ such that $Q_s = g(\hat{p}_s)$ has a solution for a given $0 \leq Q_s < g(p_s/Q_s) \leq \mathsf{E}[\theta]$. Moreover, the strict monotonicity implies that such a $\hat{p}_s$ must be unique and $\hat{p}_s > p_s/Q_s$ since $Q_s = g(\hat{p}_s) < g(p_s/Q_s)$.

At last, we show that $s_s(d_s(\bar{\theta}_s) \mid \bar{\theta}_s) > 0$. Since $\bar{\theta}_s = 0 < p_s/Q_s < \hat{p}_s$, $d_s(\bar{\theta}_s = 0) = \max\{0 - p, 0\} = 0$ By Lemma 1. Hence, $s_s(d_s(\bar{\theta}_s = 0) \mid \bar{\theta}_s = 0) = \hat{p}_s Q_s - p_s > 0$ by (4).

(b) $g(p_s/Q_s) < Q_s < \overline{Q}_s$. We first prove that $\bar{\theta}_s = 0$ when sharing with speculators emerges in equilibrium. Suppose $\bar{\theta}_s = 0$. This means type-$\bar{\theta}_s$ customers do not value the service at all. They, thus, consume nothing even though they subscribe to it. Hence, $d_s(\bar{\theta}_s) = 0$ and $s_s(d_s(\bar{\theta}_s) \mid \bar{\theta}_s) = \hat{p}_s Q_s - p_s \geq 0$, which implies $\hat{p}_s \geq p_s/Q_s$. Then, we have

$$Q_s \bar{F}(\bar{\theta}_s) = Q_s > g(p_s/Q_s) \geq g(\hat{p}_s),$$

which shows that (5) has no solution if $\bar{\theta}_s = 0$.

We first construct a pair of $(\hat{p}_s, \bar{\theta}_s)$ that satisfies (5) and $s_s(d_s(\bar{\theta}_s) \mid \bar{\theta}_s) = 0$ simultaneously. Let $\hat{p}_s = p_s/Q_s$ and we shall show that there exists a unique $\bar{\theta}_s > 0$ such that the market clearing condition (5) holds. To see this, rewrite (5)

$$Q_s \bar{F}(\bar{\theta}_s) + \hat{p}_s \bar{F}(\hat{p}_s) = \int_{\hat{p}_s}^{\Theta} \theta f(\theta) \mathrm{d}\theta \qquad Q_s \bar{F}(\bar{\theta}_s) = g(\hat{p}_s) = g(p_s/Q_s).$$

Since $g(p_s/Q_s) < Q_s$ and $\bar{F}(\theta)$ strictly decreases in θ, $Q_s \bar{F}(\bar{\theta}_s) = g(p_s/Q_s)$ holds for a unique $\bar{\theta}_s > 0$.

We next show $\bar{\theta}_s < \hat{p}_s = p_s/Q_s$. Since $Q_s \bar{F}(\bar{\theta}_s) = g(\hat{p}_s)$ and $Q_s < \overline{Q}_s = g(p_s/Q_s)/\bar{F}(p_s/Q_s)$,

$$g(\hat{p}_s) = Q_s \bar{F}(\bar{\theta}_s) < \overline{Q}_s \bar{F}(\bar{\theta}_s) = g(p_s/Q_s) \bar{F}(\bar{\theta}_s)/\bar{F}(p_s/Q_s) = g(\hat{p}_s) \bar{F}(\bar{\theta}_s)/\bar{F}(p_s/Q_s), \tag{EC.4}$$

where the last equality is due to $\hat{p}_s = p_s/Q_s$. (EC.4) implies that $\bar{F}(\bar{\theta}_s)/\bar{F}(p_s/Q_s) = \bar{F}(\bar{\theta}_s)/\bar{F}(\hat{p}_s) > 1$ and thus $\bar{\theta}_s < \hat{p}_s = p_s/Q_s$ due to the strict monotonicity of $\bar{F}(\cdot)$. Since $\bar{\theta}_s < \hat{p}_s$, $d_s(\bar{\theta}_s) = \max\{\bar{\theta}_s - p, 0\} = 0$ By Lemma 1. Hence, $s_s(d_s(\bar{\theta}_s) \mid \bar{\theta}_s) = \hat{p}_s Q_s - p_s = 0$ by (4).

So far, we have a pair of $(\hat{p}_s, \bar{\theta}_s)$ that arises as a sharing equilibrium with speculators. We next show that this is the only sharing equilibrium with speculators. First, we show that there does not exist other equilibrium with $\bar{\theta}_s > 0$. Assume there is another sharing equilibrium with speculators with $\bar{\theta}_s' = \bar{\theta}_s > 0$. From the proof of necessity, we know that for $\bar{\theta}_s' > 0$ the corresponding $\hat{p}_s'$ must equal to p_s/Q_s. However, when we set $\hat{p}_s = p_s/Q_s$ in the constructive proof above, it is shown that (EC.4) holds at a unique $\bar{\theta}_s > 0$. Therefore, we conclude $(\hat{p}_s', \bar{\theta}_s') = (\hat{p}_s, \bar{\theta}_s)$. Second, we show that there exists no equilibrium with $\bar{\theta}_s = 0$. Suppose $\bar{\theta}_s = 0$. This means type-$\bar{\theta}_s$ customers do not value the service at all. They, thus, consume nothing even though they subscribe to it. Hence, $d_s(\bar{\theta}_s) = 0$ and $s_s(d_s(\bar{\theta}_s) \mid \bar{\theta}_s) = \hat{p}_s Q_s - p_s \geq 0$, which implies $\hat{p}_s \geq p_s/Q_s$. Then, we have

$$Q_s \bar{F}(\bar{\theta}_s) = Q_s > g(p_s/Q_s) \geq g(\hat{p}_s),$$

which shows that (5) has no solution if $\bar{\theta}_s = 0$. Thus, there is no speculating equilibrium with $\bar{\theta}_s = 0$.

(ii) *Necessity.* Suppose that the sharing market has an unique equilibrium without speculators. Let $\hat{p}_s \geq 0$ be the market clearing price. By Lemma 2(ii), $\hat{p}_s \leq p_s/Q_s$, which together with the market clearing condition (6) implies

$$(Q_s + \hat{p}_s) \bar{F}(\bar{\theta}_s) = \int_{\theta_s}^{\Theta} \theta f(\theta) \mathrm{d}\theta \leq (Q_s + p_s/Q_s) \bar{F}(\bar{\theta}_s) \qquad Q_s \geq \int_{\bar{\theta}_s}^{\Theta} \theta f(\theta) \mathrm{d}\theta / \bar{F}(\bar{\theta}_s) - p_s/Q_s. \tag{EC.5}$$

The result $Q_s \geq \overline{Q}_s$ is readily available if $\int_y^{\Theta} \theta f(\theta) \mathrm{d}\theta / \bar{F}(y)$ increases in y and $\bar{\theta}_s \geq p_s/Q_s$.

To see the monotonicity of $\int_y^\Theta \theta f(\theta)\mathrm{d}\theta/\bar{F}(y)$, consider the first derivative in y

$$\left(\int_y^\Theta \theta f(\theta)\mathrm{d}\theta/\bar{F}(y)\right) = \frac{-yf(y)\bar{F}(y)+f(y)\int_y^\Theta \theta f(\theta)\mathrm{d}\theta}{\bar{F}^2(y)}.$$

By the Mean Value Theorem, there exists $\bar{y}>y$ such that

$$\left(\int_y^\Theta \theta f(\theta)\mathrm{d}\theta/\bar{F}(y)\right) = \frac{-yf(y)\bar{F}(y)+f(y)\bar{y}\int_y^\Theta f(\theta)\mathrm{d}\theta}{\bar{F}^2(y)} = \frac{-yf(y)\bar{F}(y)+\bar{y}f(y)\bar{F}(y)}{\bar{F}^2(y)}>0. \quad \text{(EC.6)}$$

To show $\bar{\theta}_s \geq p_s/Q_s$, solve $\frac{1}{2}(\bar{\theta}_s-\hat{p}_s)^2=p_s-\hat{p}_sQ_s$ and we have $Q_s+\hat{p}_s=\bar{\theta}_s \pm \sqrt{Q_s^2-2\bar{\theta}_sQ_s+2p_s}$. Note that $(Q_s+\hat{p}_s)\bar{F}(\bar{\theta}_s)=\int_{\bar{\theta}_s}^\Theta \theta f(\theta)\mathrm{d}\theta$ implies that

$$Q_s+\hat{p}_s=\int_{\bar{\theta}_s}^\Theta \theta f(\theta)\mathrm{d}\theta/\bar{F}(\bar{\theta}_s) \geq \int_{\bar{\theta}_s}^\Theta \bar{\theta}_s f(\theta)\mathrm{d}\theta/\bar{F}(\bar{\theta}_s)=\bar{\theta}_s. \quad \text{(EC.7)}$$

Thus, $Q_s+\hat{p}_s=\bar{\theta}_s+\sqrt{Q_s^2-2\bar{\theta}_sQ_s+2p_s}$. Define $l(\theta)=\theta+\sqrt{Q_s^2-2\theta Q_s+2p_s}$. Then, $\hat{p}_s=l(\bar{\theta}_s)-Q_s$. Recall that $\bar{\theta}_s \geq \hat{p}_s$ by Lemma 2(ii). Therefore,

$$\bar{\theta}_s \geq \hat{p}_s=l(\bar{\theta}_s)-Q_s \qquad \sqrt{Q_s^2-2\bar{\theta}_sQ_s+2p_s} \leq Q_s \qquad \bar{\theta}_s \geq p_s/Q_s.$$

We next prove $p_s/Q_s+Q_s/2 \leq \Theta$ by contradiction. Suppose $p_s/Q_s+Q_s/2>\Theta$. Then,

$$l(\Theta)=\Theta+\sqrt{Q_s^2-2\Theta Q_s+2p_s}>\Theta+\sqrt{Q_s^2-2Q_s\left(p_s/Q_s+Q_s/2\right)+2p_s}=\Theta.$$

Note that $\int_\Theta^\Theta \theta f(\theta)\mathrm{d}\theta/\bar{F}(\Theta)=\Theta$ due to the L'Hôpital's rule. Thus, $l(\Theta)>\int_\Theta^\Theta \theta f(\theta)\mathrm{d}\theta/\bar{F}(\Theta)$. It is easy to see that $l(\theta)$ decreases for $\theta \geq p_s/Q_s$ and recall that $\int_y^\Theta \theta f(\theta)\mathrm{d}\theta/\bar{F}(y)$ increases by (EC.6). As a result, $l(\Theta)>\int_\Theta^\Theta \theta f(\theta)\mathrm{d}\theta/\bar{F}(\Theta)$ implies that there does not exist a solution to $l(\bar{\theta}_s)=\int_{\bar{\theta}_s}^\Theta \theta f(\theta)\mathrm{d}\theta/\bar{F}(\bar{\theta}_s)$, which equivalently means that $Q_s+\hat{p}_s=\int_{\bar{\theta}_s}^\Theta \theta f(\theta)\mathrm{d}\theta/\bar{F}(\bar{\theta}_s)$ has no solution by the definition of $l(\bar{\theta}_s)$. We thus reach a contradiction.

Sufficiency. Suppose $Q_s \geq \overline{Q}_s$ and $p_s/Q_s+Q_s/2 \leq \Theta$. To ensure the existence of a sharing market without speculators with a unique market clearing price, we need to show that (6) and (EC.81) simultaneously hold with a unique set of $\hat{p}_s \geq 0$ and $\bar{\theta}_s \geq \hat{p}_s$.

Since $Q_s \geq \overline{Q}_s$, we have

$$\int_{p_s/Q_s}^\Theta \theta f(\theta)\mathrm{d}\theta/\bar{F}(p_s/Q_s) \leq Q_s+p_s/Q_s=l(p_s/Q_s). \quad \text{(EC.8)}$$

By the Mean Value Theorem, we obtain

$$\int_{p_s/Q_s+Q_s/2}^\Theta \theta f(\theta)\mathrm{d}\theta/\bar{F}(p_s/Q_s+Q_s/2)>p_s/Q_s+Q_s/2=l(p_s/Q_s+Q_s/2). \quad \text{(EC.9)}$$

Since $l(\theta)$ strictly decreases in $\theta \geq p_s/Q_s$ and $\int_{\bar{\theta}_s}^\Theta \theta f(\theta)\mathrm{d}\theta/\bar{F}(\bar{\theta}_s)$ increases, (EC.8) and (EC.9) implies that there exists a unique solution $p_s/Q_s \leq \bar{\theta}_s<p_s/Q_s+Q_s/2 \leq \Theta$ such that

$$\int_{\bar{\theta}_s}^\Theta \theta f(\theta)\mathrm{d}\theta/\bar{F}(\bar{\theta}_s)=l(\bar{\theta}_s). \quad \text{(EC.10)}$$

Furthermore, let

$$\hat{p}_s=l(\bar{\theta}_s)-Q_s=\bar{\theta}_s+\sqrt{Q_s^2-2\bar{\theta}_sQ_s+2p_s}-Q_s, \quad \text{(EC.11)}$$

which uniquely defines $\hat{p}_s$ due to the uniqueness of $\bar{\theta}_s$. Moreover, (EC.11) also suggests that

$$\hat{p}_s = \bar{\theta}_s + \sqrt{Q_s^2 - 2\bar{\theta}_s Q_s + 2p_s} - Q_s \le \bar{\theta}_s + \sqrt{Q_s^2 - 2\,(p_s/Q_s)\,Q_s + 2p_s} - Q_s = \bar{\theta}_s, \tag{EC.12}$$

Therefore, taking (EC.10) and (EC.11) together and rearranging, we claim that the following equations

$$\int_{\bar{\theta}_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta / \bar{F}(\bar{\theta}_s) = \hat{p}_s + Q_s \text{ and } \hat{p}_s = \bar{\theta}_s + \sqrt{Q_s^2 - 2\bar{\theta}_s Q_s + 2p_s} - Q_s \qquad \frac{1}{2}(\bar{\theta}_s - \hat{p}_s)^2 = p_s - \hat{p}_s Q_s,$$

must have a set of solution $\hat{p}_s$ and $\bar{\theta}_s \ge \hat{p}_s$.

At last, we show $\hat{p}_s > 0$ by contradiction. Suppose $\hat{p}_s \le 0$. According to (EC.84) and (EC.85), the discrepancy of the total supply and the demand can be written as

$$\begin{aligned}
&\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s} (Q_s - \theta + \hat{p}_s)\, f(\theta)\mathrm{d}\theta - \int_{\hat{p}_s+Q_s}^{\Theta} (\theta - Q_s - \hat{p}_s)\, f(\theta)\mathrm{d}\theta \\
=&\int_{\bar{\theta}_s}^{\Theta} Q_s f(\theta)\mathrm{d}\theta - \int_{\bar{\theta}_s}^{\Theta} (\theta - \hat{p}_s) f(\theta)\mathrm{d}\theta = \bar{F}(\bar{\theta}_s)\left(Q_s - \frac{\int_{\bar{\theta}_s}^{\Theta}(\theta - \hat{p}_s) f(\theta)\mathrm{d}\theta}{\bar{F}(\bar{\theta}_s)}\right) \\
<&\bar{F}(\bar{\theta}_s)\left(Q_s - \frac{\int_0^{\Theta}(\theta - \hat{p}_s) f(\theta)\mathrm{d}\theta}{\bar{F}(0)}\right) \le \bar{F}(\bar{\theta}_s)\left(Q_s - \frac{\int_0^{\Theta}\theta f(\theta)\mathrm{d}\theta}{\bar{F}(0)}\right) \\
=&\bar{F}(\bar{\theta}_s)\,(Q_s - \mathsf{E}[\theta]) \le 0,
\end{aligned}$$

where the first inequality results from $\displaystyle\int_y^{\Theta} \theta f(\theta)\mathrm{d}\theta / \bar{F}(y)$ strictly increases in y and the last inequality is due to Assumption 2.

Proof of Proposition 3. Proposition 2 shows that the service provider may choose (p_s, Q_s) such that either sharing with speculators or sharing without speculators takes place. We shall first show that sharing without speculators yields no less revenue than sharing with speculators.

Consider a given sharing contract (p_s, Q_s) so that $0 \le Q_s < \overline{Q}_s$. In this case, sharing with speculators occurs by Proposition 2(i). Consider the revenue function $\Pi_s(p_s, Q_s) = p_s \bar{F}(\bar{\theta}_s)$ and note that $\bar{F}(\bar{\theta}_s) = \left(\int_{\hat{p}_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta - \hat{p}_s \bar{F}(\hat{p}_s)\right)/Q_s$ by (5). Thus, we write

$$\Pi_s(p_s, Q_s) = p_s \bar{F}(\bar{\theta}_s) = p_s \left(\int_{\hat{p}_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta - \hat{p}_s \bar{F}(\hat{p}_s)\right) \Big/ Q_s. \tag{EC.13}$$

Recall $\int_y^{\Theta} \theta f(\theta)\mathrm{d}\theta - y\bar{F}(y)$ decreases in y and $\hat{p}_s \ge p_s/Q_s$ by Lemma 2(i). Therefore, (EC.13) indicates

$$\Pi_s(p_s, Q_s) \le (p_s/Q_s)\left(\int_{p_s/Q_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta - (p_s/Q_s)\bar{F}(p_s/Q_s)\right) = (p_s/Q_s)\bar{F}(p_s/Q_s)\overline{Q}_s, \tag{EC.14}$$

where $\overline{Q}_s$ is defined in Proposition 2.

We next show that there exists a sharing contract under which sharing without speculators occurs and the resulting revenue equals to $(p_s/Q_s)\bar{F}(p_s/Q_s)\overline{Q}_s$. Therefore, the maximum revenue by inducing sharing without speculators is no less than that with speculators. In particular, consider a sharing contract (p_s, Q_s) such that $p_s/Q_s = p_s/Q_s$ and $Q_s = \overline{Q}_s$, where (p_s, Q_s) is the sharing contract in the foregoing sharing with speculators case. Consider $p_s/Q_s + Q_s/2$,

$$p_s/Q_s + Q_s/2 = p_s/Q_s + \overline{Q}_s/2 = p_s/Q_s + \overline{Q}_s - \overline{Q}_s/2 = \int_{p_s/Q_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta / \bar{F}(p_s/Q_s) - \overline{Q}_s/2 \le \Theta,$$

where the third equality is due to the definition of $\overline{Q}_s$ in Proposition 2, and the inequality is due to $\int_{p_s/Q_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta / \bar{F}(p_s/Q_s) \le \Theta$ and $\overline{Q}_s/2 \ge 0$. By Proposition 2(ii), sharing without speculators occurs under (p_s, Q_s). Denote $\hat{p}_s$ as the market-clearing price and $\bar{\theta}_s$ as the subscribing threshold, i.e., $s_s\left(d_s(\bar{\theta}_s)|\bar{\theta}_s\right) = \frac{1}{2}\left(\bar{\theta}_s - \hat{p}_s\right)^2 + \hat{p}_s Q_s - p_s = 0$ under (p_s, Q_s). It can be shown that the market clearing equation (6) is achieved at $\bar{\theta}_s = \hat{p}_s$ and $\hat{p}_s = \hat{p}_s$ under (p_s, Q_s). Moreover, $\bar{\theta}_s = \hat{p}_s$ and $\hat{p}_s = \hat{p}_s$ must be the only solution due to the uniqueness of the equilibrium. Thus, $\bar{\theta}_s = \hat{p}_s$ and $s_s\left(d_s(\bar{\theta}_s)|\bar{\theta}_s\right) = \hat{p}_s Q_s - p_s = 0$, which imply $\hat{p}_s = \bar{\theta}_s = p_s/Q_s$. Recall $p_s/Q_s = p_s/Q_s$ and $Q_s = \overline{Q}_s$. Now consider the revenue under (p_s, Q_s)

$$\Pi_s(p_s, Q_s) = p_s \bar{F}(\bar{\theta}_s) = (p_s/Q_s)\bar{F}(p_s/Q_s)Q_s = (p_s/Q_s)\bar{F}(p_s/Q_s)\overline{Q}_s. \tag{EC.15}$$

Putting (EC.14) and (EC.15) together, we claim that sharing without speculators must yield a higher revenue than with speculators.

At last, we solve for the optimal contract (p_s, Q_s). Since $p_s \geq 0$ and $Q_s \geq 0$, the optimal solution is either on the boundary or a stationary point. However, $\Pi_s(p_s, Q_s) = 0$ for $p_s = 0$ or $Q_s = 0$ and $\Pi_s(p_s, Q_s) > 0$ for $p_s > 0$ and $Q_s > 0$. The optimal solution must arise at a stationary point. Consider the first-order conditions of (7)

$$\begin{cases} \dfrac{\partial \Pi_s}{\partial p_s} = \bar{F}(\bar{\theta}_s) - p_s f(\bar{\theta}_s)\dfrac{\partial \bar{\theta}_s}{\partial p_s} = 0 \\ \dfrac{\partial \Pi_s}{\partial Q_s} = -p_s f(\bar{\theta}_s)\dfrac{\partial \bar{\theta}_s}{\partial Q_s} = 0 \end{cases} \qquad \begin{cases} \dfrac{\partial \bar{\theta}_s}{\partial p_s} = \dfrac{\bar{F}(\bar{\theta}_s)}{p_s f(\bar{\theta}_s)} \\ \dfrac{\partial \bar{\theta}_s}{\partial Q_s} = 0 \end{cases}, \tag{EC.16}$$

which demonstrates the connection of the optimal contact with the subscribing threshold $\bar{\theta}_s$. We thus explore the properties of $\bar{\theta}_s$, which arises together with the market clearing price $\hat{p}_s$ via (6)

$$(Q_s + \hat{p}_s)\bar{F}(\bar{\theta}_s) = \int_{\bar{\theta}_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta \text{ and } s_s\left(d_s(\bar{\theta}_s)|\bar{\theta}_s\right) = \frac{1}{2}\left(\bar{\theta}_s - \hat{p}_s\right)^2 + \hat{p}_s Q_s - p_s = 0. \tag{EC.17}$$

Writing $\hat{p}_s$ as $\frac{\int_{\bar{\theta}_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta}{\bar{F}(\bar{\theta}_s)} - Q_s$ from the former equation and substituting it into the latter one, we are able to eliminate $\hat{p}_s$ and transform (EC.17) in terms of $\bar{\theta}_s$ only

$$\left(\bar{\theta}_s - \frac{\int_{\bar{\theta}_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta}{\bar{F}(\bar{\theta}_s)}\right)^2 = 2p_s + Q_s^2 - 2\bar{\theta}_s Q_s, \tag{EC.18}$$

or equivalently

$$\bar{\theta}_s - \frac{\int_{\bar{\theta}_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta}{\bar{F}(\bar{\theta}_s)} = -\sqrt{2p_s + Q_s^2 - 2\bar{\theta}_s Q_s} \tag{EC.19}$$

given that $\bar{\theta}_s < \frac{\int_{\bar{\theta}_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta}{\bar{F}(\bar{\theta}_s)}$ by the Mean Value Theorem. Note that $\bar{\theta}_s$ is a function of p_s and Q_s. Then, taking the first derivative of both sides of (EC.18) with respect to p_s and Q_s respectively, we have

$$\left[1 + \frac{f(\bar{\theta}_s)}{\bar{F}(\bar{\theta}_s)}\left(\bar{\theta}_s - \frac{\int_{\bar{\theta}_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta}{\bar{F}(\bar{\theta}_s)}\right)\right]\left(\bar{\theta}_s - \frac{\int_{\bar{\theta}_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta}{\bar{F}(\bar{\theta}_s)}\right)\frac{\partial \bar{\theta}_s}{\partial p_s} = 1 - \frac{\partial \bar{\theta}_s}{\partial p_s}Q_s$$

and

$$\left[1 + \frac{f(\bar{\theta}_s)}{\bar{F}(\bar{\theta}_s)}\left(\bar{\theta}_s - \frac{\int_{\bar{\theta}_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta}{\bar{F}(\bar{\theta}_s)}\right)\right]\left(\bar{\theta}_s - \frac{\int_{\bar{\theta}_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta}{\bar{F}(\bar{\theta}_s)}\right)\frac{\partial \bar{\theta}_s}{\partial Q_s} = Q_s - \bar{\theta}_s - \frac{\partial \bar{\theta}_s}{\partial Q_s}Q_s.$$

By (EC.19), the foregoing equations thus can be written as

$$\left[-\sqrt{2p_s + Q_s^2 - 2\bar{\theta}_s Q_s} + \frac{f(\bar{\theta}_s)}{\bar{F}(\bar{\theta}_s)}\left(2p_s + Q_s^2 - 2\bar{\theta}_s Q_s\right)\right]\frac{\partial \bar{\theta}_s}{\partial p_s} = 1 - \frac{\partial \bar{\theta}_s}{\partial p_s}Q_s \tag{EC.20}$$

and

$$\left[-\sqrt{2p_s+Q_s^2-2\bar{\theta}_sQ_s}+\frac{f(\bar{\theta}_s)}{\bar{F}(\bar{\theta}_s)}\left(2p_s+Q_s^2-2\bar{\theta}_sQ_s\right)\right]\frac{\partial\bar{\theta}_s}{\partial Q_s}=Q_s-\bar{\theta}_s-\frac{\partial\bar{\theta}_s}{\partial Q_s}Q_s. \tag{EC.21}$$

So far, we have (EC.16) to define the optimal contract, which is denoted as (p_s, Q_s) from now on, and eqs. (EC.20)–(EC.21) to characterize the subscribing threshold under (p_s, Q_s). We next solve (EC.16), (EC.20), and (EC.21). Substituting (EC.16) into (EC.20) and (EC.21), we obtain

$$\bar{\theta}_s=Q_s \text{ and } \frac{\bar{F}(\bar{\theta}_s)}{f(\bar{\theta}_s)}=\frac{Q_s+\sqrt{2p_s-{Q_s}^2}}{2} \qquad p_s=\frac{{Q_s}^2}{2}+\frac{1}{2}\left(\frac{2\bar{F}(Q_s)}{f(Q_s)}-Q_s\right)^2, \tag{EC.22}$$

which demonstrates that the subscribing threshold $\bar{\theta}_s$ and the optimal contract price p_s are uniquely determined by the optimal allowance Q_s. Substitute $\bar{\theta}_s=Q_s$ to (EC.19) and rearrange.

$$\frac{\int_{Q_s}^{\Theta}\theta f(\theta)\mathrm{d}\theta}{2\bar{F}(Q_s)}=\frac{\bar{\theta}_s+\sqrt{2p_s+{Q_s}^2-2\bar{\theta}_sQ_s}}{2}\overset{\text{by (EC.22)}}{=}\frac{Q_s+\sqrt{2p_s-{Q_s}^2}}{2}=\frac{\bar{F}(\bar{\theta}_s)}{f(\bar{\theta}_s)}=\frac{\bar{F}(Q_s)}{f(Q_s)},$$

which defines the optimal allowance Q_s. The market clearing price $\hat{p}_s=\sqrt{2p_s-{Q_s}^2}$ is readily obtained by solving $s_s\left(d_s(\bar{\theta}_s)|\bar{\theta}_s\right)=0$ in (EC.17) and replacing $\bar{\theta}_s$ with Q_s.

Substitute $\bar{\theta}_s=Q_s$ into $\frac{1}{2}(\bar{\theta}_s-\hat{p}_s)^2+\hat{p}_sQ_s=p_s$, it is easy to see $\hat{p}_s=\sqrt{2p_s-{Q_s}^2}$. Therefore, we know the optimal solutions in this case is that described in the Proposition 3.

At last, since $\bar{\theta}_s\geq\hat{p}_s$ by Lemma 2(ii), $d_s(\theta)=\theta-\hat{p}_s$ for all subscribers according to Lemma 1.

Proof of Proposition 4. By Lemma 4, if $p_n>\frac{1}{2}(\Theta-\hat{p}_n)^2+\hat{p}_nQ_n$, no customers subscribe and $\Pi(p_n,Q_n,\hat{p}_n)=0$. We thus only need to consider $0\leq p_n\leq\frac{1}{2}(\Theta-\hat{p}_n)^2+\hat{p}_nQ_n$. Specifically, we deliberate three cases: (a) $0\leq p_n<Q_n^2/2$; (b) $Q_n^2/2\leq p<\hat{p}_nQ_n^2+Q_n^2/2$; and (c) $\hat{p}_nQ_n^2+Q_n^2/2\leq p_n\leq\frac{1}{2}(\Theta-\hat{p}_n)^2+\hat{p}_nQ_n$. We shall show that case (c) yields more profit than the other two cases and shall characterize the optimal solutions from case (c).

(a) $0\leq p_n<Q_n^2/2$. In this case, $\bar{\theta}_n=\sqrt{2p_n}<Q_n$ by (11). The revenue function in (12) and its derivative in Q_n can be written as

$$\Pi_n(p_n,Q_n,\hat{p}_n)=p_n\bar{F}(\bar{\theta}_n)+\hat{p}_n\int_{\hat{p}_n+Q_n}^{\Theta}(\theta-\hat{p}_n-Q_n)f(\theta)\mathrm{d}\theta \text{ and } \frac{\partial\Pi_n}{\partial Q_n}=-\hat{p}_n\bar{F}(\hat{p}_n+Q_n)\leq 0.$$

It is obvious that $\Pi(p_n,Q_n,\hat{p}_n)$ is decreasing in Q_n for a given p_n if $0\leq p_n<Q_n^2/2$. Hence, $\Pi(p_n,Q_n,\hat{p}_n)<\Pi(p_n,Q_n=\sqrt{2p_n},\hat{p}_n)$ in this case. Therefore, the profit cannot be more than that when $\frac{1}{2}Q_n^2\leq p_n<\hat{p}_nQ_n+\frac{1}{2}Q_n^2$, i.e., case (b).

(b) $Q_n^2/2\leq p_n<\hat{p}_nQ_n+Q_n^2/2$. In this case, $\bar{\theta}_n=p_n/Q_n+Q_n/2$ by (11) and $Q_n\leq\bar{\theta}_n<\hat{p}_n+Q_n$. We can rewrite the revenue function $\Pi_n(p_n,Q_n,\hat{p}_n)$ in (12) in terms of $(\bar{\theta}_n,Q_n,\hat{p}_n)$ as

$$\Pi_n(\bar{\theta}_n,Q_n,\hat{p}_n)=(\bar{\theta}_nQ_n-\frac{1}{2}Q_n^2)\bar{F}(\bar{\theta}_n)+\hat{p}_n\int_{\hat{p}_n+Q_n}^{\Theta}(\theta-\hat{p}_n-Q_n)f(\theta)\mathrm{d}\theta.$$

We shall show that the maximum value of $\Pi_n(\bar{\theta}_n,Q_n,\hat{p}_n)$ must be achieved at $\bar{\theta}_n=\hat{p}_n+Q_n$. In other words, the optimal solution must be a boundary point, which will be considered in case (c)

Assume that there is an interior solution $(\bar{\theta}_n,Q_n,\hat{p}_n)$, which must satisfy the following FOCs

$$\begin{cases} \dfrac{\partial \Pi_n}{\partial \bar{\theta}_n} = Q_n \cdot \Big(\bar{F}(\bar{\theta}_n) - (\bar{\theta}_n - Q_n/2) f(\bar{\theta}_n)\Big) = 0, & \text{(EC.23)} \\ \dfrac{\partial \Pi_n}{\partial \hat{p}_n} = \displaystyle\int_{\hat{p}_n + Q_n}^{\Theta} \theta f(\theta) \mathrm{d}\theta - (2\hat{p}_n + Q_n)\bar{F}(\hat{p}_n + Q_n) = 0, & \text{(EC.24)} \\ \dfrac{\partial \Pi_n}{\partial Q_n} = (\bar{\theta}_n - Q_n)\bar{F}(\theta_n) - \hat{p}_n \bar{F}(\hat{p}_n + Q_n) = 0. & \text{(EC.25)} \end{cases}$$

Define an auxiliary function $h(x \mid Q_n) = (x - Q_n)\bar{F}(x)$ for $x \geq 0$ and denote x_0 as the solution to

$$\frac{\partial h}{\partial x} = \bar{F}(x) - (x - Q_n) f(x) = x f(x) \left(\frac{\bar{F}(x)}{x f(x)} - \left(1 - \frac{Q_n}{x}\right) \right) = 0. \tag{EC.26}$$

Rewrite (EC.25) in terms of the auxiliary function $h(\cdot \mid Q_n)$ as

$$\frac{\partial \Pi_n}{\partial Q_n} = h(\bar{\theta}_n \mid Q_n) - h(\hat{p}_n + Q_n \mid Q_n) = 0. \tag{EC.27}$$

We shall show that (EC.27) can only hold at $\bar{\theta}_n = \hat{p}_n + Q_n$ in three steps:

Step 1. Prove that $h(x \mid Q_n)$ strictly increases in $x \leq x_0$ and strictly decreases in $x \geq x_0$;

Step 2. Show that $\hat{p}_n + Q_n \leq x_0$;

Step 3. Show that $\bar{\theta}_n < x_0$.

Steps 2 and 3 provide a range of the interior optimal solution. And Step 1 shows that the auxiliary function $h(x \mid Q_n)$ has a strict monotonicity in this range. Thus, (EC.27) can only hold at $\bar{\theta}_n = \hat{p}_n + Q_n$, which contradicts with the assumption that $(\bar{\theta}_n, Q_n, \hat{p}_n)$ is an interior optimal solution.

We next demonstrate the claims in the three steps.

Step 1. Recall that $F(\cdot)$ has an IFR. Then, $\frac{\bar{F}(x)}{x f(x)} \quad [0, \infty)$ is decreasing in x. On the other hand, $1 - Q_n/x \quad (-\infty, 1]$ is strictly increasing in x. Thus, x_0 be the unique solution to (EC.26). The monotonicity of $h(x \mid Q_n)$ follows because

$$\frac{\bar{F}(x)}{x f(x)} > (<) \left(1 - \frac{Q_n}{x}\right) \text{ iff } \ x < (>)\ x_0 \qquad\qquad \frac{\partial h}{\partial x} > (<) 0 \text{ iff } \ x < (>)\ x_0. \tag{EC.28}$$

Step 2. For convenience, define a dummy variable $z = \hat{p}_n + Q_n$. Rewrite (EC.24) in terms of z,

$$\begin{aligned} \frac{\partial \Pi_n}{\partial \hat{p}_n} &= \int_z^{\Theta} \theta f(\theta) \mathrm{d}\theta - (2z - Q_n)\bar{F}(z) \\ &= \int_z^{\Theta} (\theta - z) f(\theta) \mathrm{d}\theta - (z - Q_n)\bar{F}(z) \\ \text{(integration by parts)} &= \int_z^{\Theta} \bar{F}(z) \mathrm{d}\theta - (z - Q_n)\bar{F}(z) \\ &= z\bar{F}(z) \left(\frac{\int_z^{\Theta} \bar{F}(\theta) \mathrm{d}\theta}{z\bar{F}(z)} - \left(1 - \frac{Q_n}{z}\right) \right) = 0. \end{aligned} \tag{EC.29}$$

By Assumption 1, $\frac{\int_x^{\Theta} \bar{F}(x) \mathrm{d}\theta}{x\bar{F}(x)} \quad [0, \infty)$ is decreasing in x. Moreover, $1 - Q_n/x \quad [-\infty, 1)$ is strictly increasing in x. Thus, z is the unique solution to (EC.29). Note that $\frac{\int_x^{\Theta} \bar{F}(\theta) \mathrm{d}\theta}{x\bar{F}(x)} \leq \frac{\bar{F}(x)}{x f(x)}$ by Assumption 1, thus $z = \hat{p}_n + Q_n \leq x_0$.

Step 3. Recall that case (b) only focuses on the case that $Q_n^2/2 \leq p_n < \hat{p}_n Q_n + Q_n^2/2$. Thus, $Q_n = 0$ is not valid consideration. (In fact, it is a special situation of case (c).) As a result, we have

$$\frac{\partial \Pi_n}{\partial \bar{\theta}_n} = 0 \qquad \bar{F}(\bar{\theta}_n) = (\bar{\theta}_n - Q_n/2) f(\bar{\theta}_n) > (\bar{\theta}_n - Q_n) f(\bar{\theta}_n) = \quad \frac{\bar{F}(\bar{\theta}_n)}{\bar{\theta}_n f(\bar{\theta}_n)} > 1 - \frac{Q_n}{\bar{\theta}_n}.$$

By (EC.28), we conclude $\bar{\theta}_n < x_0$.

(c) $\hat{p}_n Q_n + Q_n^2/2 \le p_n \le \frac{1}{2}(\Theta - \hat{p}_n)^2 + \hat{p}_n Q_n$. By (11), we have $\bar{\theta}_n = \hat{p}_n + \sqrt{2(p_n - \hat{p}_n Q_n)}$, which is derived from setting (EC.92) to zero so that $\frac{1}{2}(\bar{\theta}_n - \hat{p}_n)^2 = p_n - \hat{p}_n Q_n$. Rewrite the profit function (12),

$$\Pi_n(p_n, Q_n, \hat{p}_n) = (\frac{1}{2}\bar{\theta}_n^2 - \bar{\theta}_n \hat{p}_n - \frac{1}{2}\hat{p}_n^2)\bar{F}(\bar{\theta}_n) + \hat{p}_n \int_{\bar{\theta}_n}^{\Theta} \theta f(\theta) \mathrm{d}\theta, \tag{EC.30}$$

which only depends on $\bar{\theta}_n$ and $\hat{p}_n$. By the FOCs, the optimal nonlinear contract must satisfy

$$\begin{cases} \dfrac{\partial \Pi_n}{\partial \bar{\theta}_n} = (\bar{\theta}_n - \hat{p}_n)\left(\bar{F}(\bar{\theta}_n) - \dfrac{1}{2}(\bar{\theta}_n + \hat{p}_n) f(\bar{\theta}_n)\right) = 0, & \text{(EC.31)} \\ \dfrac{\partial \Pi_n}{\partial \hat{p}_n} = \displaystyle\int_{\bar{\theta}_n}^{\Theta} \theta f(\theta) \mathrm{d}\theta - (\bar{\theta}_n + \hat{p}_n)\bar{F}(\bar{\theta}_n) = 0. & \text{(EC.32)} \end{cases}$$

We consider the solutions to (EC.31) and (EC.32) for two situations: (c1) $\bar{\theta}_n > \hat{p}_n$ and (c2) $\bar{\theta}_n = \hat{p}_n$.

(c1) If $\bar{\theta}_n > \hat{p}_n$, the FOCs (EC.31) and (EC.32) become

$$\begin{cases} \dfrac{\bar{F}(\bar{\theta}_n)}{f(\bar{\theta}_n)} = \dfrac{1}{2}(\bar{\theta}_n + \hat{p}_n), & \text{(EC.33)} \\ \dfrac{\int_{\bar{\theta}_n}^{\Theta} \theta f(\theta) \mathrm{d}\theta}{\bar{F}(\bar{\theta}_n)} = \bar{\theta}_n + \hat{p}_n. & \text{(EC.34)} \end{cases}$$

Eliminating $\hat{p}_n$ from (EC.33) and (EC.34), the optimal $\bar{\theta}_n$ solves

$$\frac{\displaystyle\int_{\bar{\theta}_n}^{\Theta} \theta f(\theta) \mathrm{d}\theta}{2\bar{F}(\bar{\theta}_n)} = \frac{\bar{F}(\bar{\theta}_n)}{f(\bar{\theta}_n)}.$$

The optimal $\hat{p}_n$ and Q_n are obtained by (EC.32) and $\frac{1}{2}(\bar{\theta}_n - \hat{p}_n)^2 = p_n - \hat{p}_n Q_n$ in (EC.92), respectively.

(c2) If $\bar{\theta}_n - \hat{p}_n = 0$, then $p_n = \hat{p}_n Q_n$ by (11). Recall that in case (c) $p_n \ge \hat{p}_n Q_n + Q_n^2/2$. Thus, $p_n = \hat{p}_n Q_n \ge \hat{p}_n Q_n + Q_n^2/2$, which implies $Q_n = 0$ and $p_n = 0$. Now solve (EC.32) and integrate by parts,

$$\bar{\theta}_n + \hat{p}_n = \frac{\int_{\bar{\theta}_n}^{\Theta} \theta f(\theta) \mathrm{d}\theta}{\bar{F}(\bar{\theta}_n)} \qquad \bar{\theta}_n = \frac{1}{2}(\bar{\theta}_n + \hat{p}_n) = \frac{\int_{\bar{\theta}_n}^{\Theta} \theta f(\theta) \mathrm{d}\theta}{2\bar{F}(\bar{\theta}_n)} = \frac{\int_{\bar{\theta}_n}^{\Theta} \bar{F}(\theta) \mathrm{d}\theta + \bar{\theta}_n \bar{F}(\bar{\theta}_n)}{2\bar{F}(\bar{\theta}_n)}, \tag{EC.35}$$

Note that $\bar{\theta}_n = \dfrac{\int_{\bar{\theta}_n}^{\Theta} \bar{F}(\theta) \mathrm{d}\theta + \bar{\theta}_n \bar{F}(\bar{\theta}_n)}{2\bar{F}(\bar{\theta}_n)}$ indicates $\int_{\bar{\theta}_n}^{\Theta} \bar{F}(\theta) \mathrm{d}\theta = \bar{\theta}_n \bar{F}(\bar{\theta}_n)$. Thus, we can rewrite (EC.35) as

$$\bar{\theta}_n = \frac{1}{2}(\bar{\theta}_n + \hat{p}_n) = \frac{\int_{\bar{\theta}_n}^{\Theta} \theta f(\theta) \mathrm{d}\theta}{2\bar{F}(\bar{\theta}_n)} = \frac{\int_{\bar{\theta}_n}^{\Theta} \bar{F}(\theta) \mathrm{d}\theta + \bar{\theta}_n \bar{F}(\bar{\theta}_n)}{2\bar{F}(\bar{\theta}_n)} = \frac{\int_{\bar{\theta}_n}^{\Theta} \bar{F}(\theta) \mathrm{d}\theta}{\bar{F}(\bar{\theta}_n)} \le \frac{\bar{F}(\bar{\theta}_n)}{f(\bar{\theta}_n)}, \tag{EC.36}$$

where the inequality stems from Assumption 1.

We next show that the inequality in (EC.36) has to be binding at optimality. Assume that $\bar{\theta}_n < \frac{\bar{F}(\bar{\theta}_n)}{f(\bar{\theta}_n)}$ and consider the marginal profit at $\bar{\theta}_n + \epsilon$ with a slight abuse of notation, where $\epsilon > 0$ such that $\frac{\bar{F}(\bar{\theta}_n + \epsilon)}{(\bar{\theta}_n + \epsilon) f(\bar{\theta}_n + \epsilon)} > 1$,

$$\begin{aligned} \frac{\partial \Pi_n}{\partial \bar{\theta}_n}\Big|_{\bar{\theta}_n = \bar{\theta}_n + \epsilon} &= (\bar{\theta}_n - \hat{p}_n)\left[\bar{F}(\bar{\theta}_n) - \frac{1}{2}(\bar{\theta}_n + \hat{p}_n) f(\bar{\theta}_n)\right]\Big|_{\bar{\theta}_n = \bar{\theta}_n + \epsilon} \\ &= \epsilon \frac{f(\bar{\theta}_n + \epsilon)}{\bar{\theta}_n + \epsilon}\left[\frac{\bar{F}(\bar{\theta}_n + \epsilon)}{(\bar{\theta}_n + \epsilon) f(\bar{\theta}_n + \epsilon)} - \frac{1}{2}(1 + \frac{\hat{p}_n}{\bar{\theta}_n + \epsilon})\right] > 0, \end{aligned} \tag{EC.37}$$

where the last equality and inequality are both due to $\bar{\theta}_n = \hat{p}_n$. (EC.37) implies that in the right neighborhood of $\bar{\theta}_n$ there exists a $\bar{\theta}_n+\epsilon$ that yields a higher profit, which contradicts with $\bar{\theta}_n$ is the optimal solution. Therefore, the inequality of (EC.36) has to be binding and the optimal $\bar{\theta}_n$ solves

$$\frac{\int_{\bar{\theta}_n}^{\Theta}\theta f(\theta)\mathrm{d}\theta}{2\bar{F}(\bar{\theta}_n)}=\frac{\bar{F}(\bar{\theta}_n)}{f(\bar{\theta}_n)}.$$

The optimal $\hat{p}_n$ and Q_n are obtained by (EC.32) and $\frac{1}{2}(\bar{\theta}_n-\hat{p}_n)^2=p_n-\hat{p}_nQ_n$ in (EC.92), respectively.

Proof of Theorem 1. By Propositions 3 and 4, we have

$$\frac{\int_{Q_s}^{\Theta}\theta f(\theta)\mathrm{d}\theta}{2\bar{F}(Q_s)}=\frac{\bar{F}(Q_s)}{f(Q_s)}\quad \textit{and}\quad \frac{\int_{\bar{\theta}_n}^{\Theta}\theta f(\theta)\mathrm{d}\theta}{2\bar{F}(\bar{\theta}_n)}=\frac{\bar{F}(\bar{\theta}_n)}{f(\bar{\theta}_n)},$$

respectively. Thus $Q_s=\bar{\theta}_n$. Recall that $\bar{\theta}_s=Q_s$ from Proposition 3. Hence, $\bar{\theta}_s=\bar{\theta}_n$.

Now we show that $\hat{p}_s=\hat{p}_n$ as presented in Proposition 5(ii). Recall from (EC.17) that the optimal sharing contract satisfies $s_s\left(d_s(\bar{\theta}_s)|\bar{\theta}_s\right)=\frac{1}{2}\left(\bar{\theta}_s-\hat{p}_s\right)^2+\hat{p}_sQ_s-p_s=0$, which can be written as

$$\frac{1}{2}({Q_s}^2+{\hat{p}_s}^2)=p_s, \tag{EC.38}$$

since $Q_s=\bar{\theta}_n$. Moreover, Proposition 3 indicates that $\frac{\bar{F}(Q_s)}{f(Q_s)}=\frac{Q_s+\sqrt{2p_s-{Q_s}^2}}{2}$, which is equivalent to $\frac{\bar{F}(Q_s)}{f(Q_s)}=\frac{Q_s+\hat{p}_s}{2}$ by (EC.38). Note that Proposition 4 reveals that under the optimal nonlinear contract $\frac{\bar{F}(\bar{\theta}_n)}{f(\bar{\theta}_n)}=\frac{\bar{\theta}_n+\hat{p}_n}{2}$. Since $Q_s=\bar{\theta}_n$, then $\hat{p}_s=\hat{p}_n$.

At last, consider the optimal revenues of the two contracts. By (EC.38), we can derive the optimal sharing contract's revenue as

$$\Pi_s(p_s,Q_s)=p_s\bar{F}(\bar{\theta}_s)=\frac{1}{2}({Q_s}^2+{\hat{p}_s}^2)\bar{F}(\bar{\theta}_s)=\frac{1}{2}({Q_s}^2+{\hat{p}_s}^2)\bar{F}(Q_s).$$

On the other hand, by (EC.30), the optimal nonlinear contract's revenue can be written as

$$\begin{aligned}\Pi_n(p_n,Q_n,\hat{p}_n)&=(\frac{1}{2}{\bar{\theta}_n}^2-\bar{\theta}_n\hat{p}_n-\frac{1}{2}{\hat{p}_n}^2)\bar{F}(\bar{\theta}_n)+\hat{p}_n\int_{\bar{\theta}_n}^{\Theta}\theta f(\theta)\mathrm{d}\theta\\&=(\frac{1}{2}{\bar{\theta}_n}^2-\bar{\theta}_n\hat{p}_n-\frac{1}{2}{\hat{p}_n}^2)\bar{F}(\bar{\theta}_n)+\hat{p}_n(\hat{p}_n+\bar{\theta}_n)\bar{F}(\bar{\theta}_n)\\&=\frac{1}{2}({\bar{\theta}_n}^2+{\hat{p}_n}^2)\bar{F}(\bar{\theta}_n),\end{aligned}$$

where the second equality is due to $\frac{\int_{\bar{\theta}_n}^{\Theta}\theta f(\theta)\mathrm{d}\theta}{\bar{F}(\bar{\theta}_n)}=\hat{p}_n+\bar{\theta}_n$ from Proposition 4. Since $Q_s=\bar{\theta}_n$ and $\hat{p}_s=\hat{p}_n$, thus $\Pi_s(p_s,Q_s)=\Pi_n(p_n,Q_n,\hat{p}_n)$.

Proof of Proposition 5. Note that we have already proved $\hat{p}_s=\hat{p}_n$ in the proof of Theorem 1. We now will use this equality to demonstrate other results in this proposition.

(i) By (7), the optimal sharing contract's revenue $\Pi_s(p_s,Q_s)=p_s\bar{F}(\bar{\theta}_s)$. Using (13), we can write the nonlinear contract's revenue (12) at optimality as

$$\Pi_n(p_n,Q_n,\hat{p}_n)=p_n\bar{F}(\bar{\theta}_n)+\hat{p}_n\int_{\hat{p}_n+Q_n}^{\Theta}(\theta-\hat{p}_n)f(\theta)\mathrm{d}\theta. \tag{EC.39}$$

Recall from Theorem 1 that $\Pi_s(p_s,Q_s)=\Pi_n(p_n,Q_n,\hat{p}_n)$. Thus, $p_s\bar{F}(\bar{\theta}_s)>p_n\bar{F}(\bar{\theta}_n)$. Moreover, since Theorem 1 also shows that $\bar{\theta}_s=\bar{\theta}_n$, we have $p_s>p_n$. To see $Q_s>Q_n$, recall that $\bar{\theta}_n\geq Q_n+\hat{p}_n$ by Corollary 2 and $\bar{\theta}_s<\hat{p}_s+Q_s$ by Corollary 1. Since $\bar{\theta}_s=\bar{\theta}_n$ by Theorem 1 and $\hat{p}_s=\hat{p}_n$, then $Q_s>Q_n$.

(ii) The total customer consumptions under the optimal sharing and nonlinear contracts are

$$Q_s\bar{F}(\bar{\theta}_s) \quad \text{and} \quad Q_n\bar{F}(\bar{\theta}_n) + \int_{\hat{p}_n+Q_n}^{\Theta} (\theta - \hat{p}_n) f(\theta)\mathrm{d}\theta,$$

respectively. Since $\bar{\theta}_s = \bar{\theta}_n$ by Theorem 1 and subscribers consume the same amount of data under two contracts that will be shown in Theorem 2, it must be the total consumptions are the same under sharing and nonlinear contracts i.e.,

$$Q_s\bar{F}(\bar{\theta}_s) = Q_n\bar{F}(\bar{\theta}_n) + \int_{\hat{p}_n+Q_n}^{\Theta} (\theta - \hat{p}_n) f(\theta)\mathrm{d}\theta \qquad Q_s\bar{F}(\bar{\theta}_s) - Q_n\bar{F}(\bar{\theta}_n) = \int_{\hat{p}_n+Q_n}^{\Theta} (\theta - \hat{p}_n) f(\theta)\mathrm{d}\theta. \tag{EC.40}$$

The optimal nonlinear contract's revenue in (EC.39) thus can be written as

$$\Pi_n(p_n, Q_n, \hat{p}_n) = p_n\bar{F}(\bar{\theta}_n) + \hat{p}_n \int_{\hat{p}_n+Q_n}^{\Theta} (\theta - \hat{p}_n) f(\theta)\mathrm{d}\theta = p_n\bar{F}(\bar{\theta}_n) + \hat{p}_n(Q_s\bar{F}(\bar{\theta}_s) - Q_n\bar{F}(\bar{\theta}_n)).$$

Since $\Pi_n(p_n, Q_n, \hat{p}_n) = \Pi_s(p_s, Q_s) = p_s\bar{F}(\bar{\theta}_s)$ and $\bar{\theta}_s = \bar{\theta}_n$ by Theorem 1, therefore

$$\Pi_s(p_s, Q_s) - \Pi_n(p_n, Q_n, \hat{p}_n) = p_s\bar{F}(\bar{\theta}_s) - p_n\bar{F}(\bar{\theta}_n) - \hat{p}_n(Q_s\bar{F}(\bar{\theta}_s) - Q_n\bar{F}(\bar{\theta}_n)) = 0 \qquad p_s = p_n + \hat{p}_n(Q_s - Q_n).$$

(iii) We prove $\hat{p}_n \le p_s/Q_s \le p_n/Q_n$ by contradiction. First, suppose $p_s/Q_s > p_n/Q_n$. By Proposition 4, $p_n - \hat{p}_f Q_n = \frac{1}{2}(\bar{\theta}_n - \hat{p}_n)^2 \ge 0$. Then, $p_s/Q_s > p_n/Q_n \ge \hat{p}_n$. Therefore, we have

$$\begin{aligned}
\Pi_n(p_n, Q_n, \hat{p}_n) &= p_n\bar{F}(\bar{\theta}_n) + \hat{p}_n \int_{\hat{p}_n+Q_n}^{\Theta} (\theta - \hat{p}_n) f(\theta)\mathrm{d}\theta \\
&= \frac{p_n}{Q_n} Q_n\bar{F}(\bar{\theta}_n) + \hat{p}_n \int_{\hat{p}_n+Q_n}^{\Theta} (\theta - \hat{p}_n) f(\theta)\mathrm{d}\theta \\
&< \frac{p_s}{Q_s} Q_n\bar{F}(\bar{\theta}_n) + \frac{p_s}{Q_s} \int_{\hat{p}_n+Q_n}^{\Theta} (\theta - \hat{p}_n) f(\theta)\mathrm{d}\theta \\
&= \frac{p_s}{Q_s} Q_s\bar{F}(\bar{\theta}_s) = \Pi_s(p_s, Q_s),
\end{aligned}$$

where the third equality is due to (EC.40). However, $\Pi_n(p_n, Q_n, \hat{p}_n) < \Pi_s(p_s, Q_s)$ contradicts with $\Pi_n(p_n, Q_n, \hat{p}_n) = \Pi_s(p_s, Q_s)$ as shown in Theorem 1. Thus, $p_s/Q_s \le p_n/Q_n$.

Second, suppose $\hat{p}_n > p_s/Q_s$. Since we have demonstrated that $p_n/Q_n \ge p_s/Q_s$ above, then

$$\begin{aligned}
\Pi_n(p_n, Q_n, \hat{p}_n) &= p_n\bar{F}(\bar{\theta}_n) + \hat{p}_n \int_{\hat{p}_n+Q_n}^{\Theta} (\theta - \hat{p}_n) f(\theta)\mathrm{d}\theta \\
&= \frac{p_n}{Q_n} Q_n\bar{F}(\bar{\theta}_n) + \hat{p}_n \int_{\hat{p}_n+Q_n}^{\Theta} (\theta - \hat{p}_n) f(\theta)\mathrm{d}\theta \\
&> \frac{p_s}{Q_s} Q_n\bar{F}(\bar{\theta}_n) + \frac{p_s}{Q_s} \int_{\hat{p}_n+Q_n}^{\Theta} (\theta - \hat{p}_n) f(\theta)\mathrm{d}\theta \\
&= \frac{p_s}{Q_s} Q_s\bar{F}(\bar{\theta}_s) = \Pi_s(p_s, Q_s),
\end{aligned}$$

which contradicts with $\Pi_n(p_n, Q_n, \hat{p}_n) = \Pi_s(p_s, Q_s)$.

Proof of Theorem 2. (i) For a type-θ subscriber, where $\theta \ge \bar{\theta}_s = \bar{\theta}_n$, Proposition 3(iii) and Corollary 2 tell us that her demands under sharing and nonlinear contracts are $d_s(\theta) = \theta - \hat{p}_s$ and $d_n(\theta) = \theta - \hat{p}_n$, respectively. Since $\hat{p}_s = \hat{p}_n$ by Proposition 5(ii), we have $d_s(\theta) = d_n(\theta)$.

(ii) Note $d_s(\theta) = \theta - \hat{p}_s$ and $d_n(\theta) = \theta - \hat{p}_n$, according to (4) and (9), we can write the consumer surplus under sharing and nonlinear contracts as

$$s_s(d_s(\theta) \mid \theta) = \frac{1}{2}(\theta - \hat{p}_s)^2 - (p_s - \hat{p}_s Q_s) = \frac{1}{2}(\theta - \hat{p}_s)^2 - \frac{1}{2}(\bar{\theta}_s - \hat{p}_s)^2$$

and

$$s_n(d_n(\theta) \mid \theta) = \frac{1}{2}(\theta - \hat{p}_n)^2 - (p_n - \hat{p}_n Q_n),$$

respectively. Note that $\frac{1}{2}(\bar{\theta}_n - \hat{p}_n)^2 = p_n - \hat{p}_n Q_n$ by Proposition 4(ii), $\hat{p}_s = \hat{p}_n$ by Proposition 5(ii), and $\bar{\theta}_s = \bar{\theta}_n$ by Theorem 1(ii). Therefore, $s_s(d_s(\theta) \mid \theta) = s_n(d_n(\theta) \mid \theta)$.

Proof of Theorem 3. First, let $\{p_{s_k}, Q_{s_k}\}$, $k = 1, 2 \ldots, K$, be the optimal menu of the sharing contract. By Lemma TS4, we can focus on the case that there are no speculators under this optimal sharing menu. Moreover, by Lemma TS3, subscribers must be divided into K consecutive intervals $[\bar{\theta}_k, \bar{\theta}_{k+1})$, where $k = 1, 2 \ldots, K$ and $\bar{\theta}_{K+1} = \Theta$. Now construct a K-tier menu of the nonlinear contract such that $\{p_{n_k} = p_{s_k} - \hat{p}_{s_k} Q_{s_k}, Q_{n_k} = 0, \hat{p}_{n_k} = \hat{p}_{s_k}\}$. Then, for any $\theta \in [\bar{\theta}_k, \bar{\theta}_{k+1})$, Theorems 1 and 2 hold. Thus, the two contracts yield the same outcome.

Similarly, if $\{p_{n_k}, Q_{n_k}, \hat{p}_{n_k}\}$, $k = 1, 2 \ldots, K$, is the optimal K-tier menu of the nonlinear contracts, we can construct a menu of sharing contracts $\{p_{s_k}, Q_{s_k}\}$ that yield the same outcome, where p_{s_k} and Q_{s_k} are given by

$$\begin{cases} p_{s_k} = p_{n_k} + \hat{p}_{n_k}(Q_{s_k} - Q_{n_k}), \\ (Q_{s_k} + \hat{p}_{n_k})[\bar{F}(\bar{\theta}_k) - \bar{F}(\bar{\theta}_{k+1})] = \int_{\bar{\theta}_k}^{\bar{\theta}_{k+1}} \theta f(\theta) \mathrm{d}\theta. \end{cases}$$

Proof of Proposition 6. We show the revenue equivalence by demonstrating $\overline{\Pi}_s \leq \overline{\Pi}_n^0$ and $\overline{\Pi}_s \geq \overline{\Pi}_n^0$ hold simultaneously.

First, we prove $\overline{\Pi}_s \leq \overline{\Pi}_n^0$, where $\overline{\Pi}_n^0$ represents the optimal revenue when $Q_n = 0$. Denote (p_s, Q_s) as the optimal sharing contract. According to the Law of Large Numbers, the market clearing price $\hat{p}_s$ under (p_s, Q_s) is determined by

$$\overbrace{\sum_{\theta \in \Theta_s} Q_s}^{\text{total data supply}} = \overbrace{\sum_{\theta \in \Theta_s} \underbrace{\mathsf{P}(\theta + \epsilon_\theta - \hat{p}_s \geq 0) \mathsf{E}_{\epsilon_\theta} [d_s(\theta + \epsilon_\theta) \mid \theta + \epsilon_\theta - \hat{p}_s \geq 0]}_{\text{each subscriber's expected data demand}}}^{\text{total data demand}} \tag{EC.41a}$$

$$= \sum_{\theta \in \Theta_s} \mathsf{P}(\theta + \epsilon_\theta - \hat{p}_s \geq 0) \mathsf{E}_{\epsilon_\theta} [\theta + \epsilon_\theta - \hat{p}_s \mid \theta + \epsilon_\theta - \hat{p}_s \geq 0], \tag{EC.41b}$$

where $\Theta_s = \Theta_s(p_s, Q_s, \hat{p}_s) = \{\theta \mid \bar{s}_s(p_s, Q_s, \hat{p}_s \mid \theta) \geq 0\}$. Next, we construct a nonlinear contract with $Q_n = 0$ that yields the same revenue as the optimal sharing contract (p_s, Q_s). Consider the nonlinear contract $(p_n, Q_n, \hat{p}_n) = (p_s - \hat{p}_s Q_s, 0, \hat{p}_s)$ and the expected surplus of a type-θ customer subscribing to this contract. By (TS.12),

$$\begin{aligned} \bar{s}_n(p_n, Q_n, \hat{p}_n \mid \theta) &= \mathsf{P}(\hat{p}_n + Q_n \leq \theta + \epsilon_\theta) \mathsf{E}_{\epsilon_\theta} \left[\frac{1}{2}(\theta + \epsilon_\theta - \hat{p}_n)^2 + \hat{p}_n Q_n \middle| \hat{p}_n + Q_n \leq \theta + \epsilon_\theta \right] - p_n \\ &= \mathsf{P}(\hat{p}_n \leq \theta + \epsilon_\theta) \mathsf{E}_{\epsilon_\theta} \left[\frac{1}{2}(\theta + \epsilon_\theta - \hat{p}_n)^2 \middle| \hat{p}_n \leq \theta + \epsilon_\theta \right] - p_s + \hat{p}_s Q_s \\ &= \mathsf{P}(\hat{p}_s \leq \theta + \epsilon_\theta) \mathsf{E}_{\epsilon_\theta} \left[\frac{1}{2}(\theta + \epsilon_\theta - \hat{p}_s)^2 \middle| \hat{p}_s \leq \theta + \epsilon_\theta \right] - p_s + \hat{p}_s Q_s \\ &\overset{\text{by (21)}}{=} \bar{s}_s(p_s, Q_s, \hat{p}_s \mid \theta), \end{aligned} \tag{EC.42}$$

which implies that the constructed nonlinear contract $(p_n, Q_n, \hat{p}_n)$ has the same subscribers as the optimal sharing contract (p_s, Q_s), i.e., $\Theta_s = \Theta_n^0 := \{\theta \mid \bar{s}_n(p_n, Q_n, \hat{p}_n \mid \theta) \geq 0\}$.

Next, we compare the revenues of these two contracts. For the sharing contract, the only revenue comes from p_s that all subscribers pay. We then write $\overline{\Pi}_s = \sum_{\theta \in \Theta_s} p_s$. Since $p_n = p_s - \hat{p}_s Q_s$, then

$$\overline{\Pi}_s = \sum_{\theta \in \Theta_s} (p_n + \hat{p}_s Q_s) = \sum_{\theta \in \Theta_s} p_n + \hat{p}_s \sum_{\theta \in \Theta_s} \mathsf{P}(\theta + \epsilon_\theta - \hat{p}_s \geq 0) \mathsf{E}_{\epsilon_\theta} \left[\theta + \epsilon_\theta - \hat{p}_s \mid \theta + \epsilon_\theta - \hat{p}_s \geq 0 \right]. \quad \text{(EC.43)}$$

The nonlinear contract, on the other hand, has two revenue streams: the base subscription fee p_n and the variable payment at a rate of $\hat{p}_n$. Since $Q_n = 0$, there is no allowance and each subscriber with a strictly positive demand has to incur a variable payment. Let $\overline{\Pi}_n^0$ be the revenue of the nonlinear contract $(p_n, Q_n, \hat{p}_n)$.

$$\begin{aligned}
\overline{\Pi}_n^0 &= \sum_{\Theta_n^0} p_n + \hat{p}_n \sum_{\Theta_n^0} \mathsf{E}_{\epsilon_\theta} \left[d_s(\theta + \epsilon_\theta) \right] \\
&\overset{\text{by (20)}}{=} \sum_{\Theta_n^0} p_n + \hat{p}_n \sum_{\Theta_n^0} \mathsf{P}(\theta + \epsilon_\theta - \hat{p}_n \geq 0) \mathsf{E}_{\epsilon_\theta} \left[\theta + \epsilon_\theta - \hat{p}_n \mid \theta + \epsilon_\theta - \hat{p}_n \geq 0 \right]. \qquad \text{(EC.44)}
\end{aligned}$$

Recall that $\Theta_s = \Theta_n^0$ and $\hat{p}_s = \hat{p}_n$. Therefore, Eqs. (EC.43) and (EC.44) together implies $\overline{\Pi}_s = \overline{\Pi}_n^0 \leq \overline{\Pi}_n^0$.

Second, we prove $\overline{\Pi}_s \geq \overline{\Pi}_n^0$. Let p_n and $\hat{p}_n$ be the optimal base price and marginal rate of the nonlinear contact when $Q_n = 0$. Consider a sharing contract (p_s, Q_s) where $p_s = p_n + \hat{p}_n Q_s$. Such a sharing contract is valid if one of its parameters, either p_s or Q_s, is well defined. We choose to set Q_s. Note that the market clearing equation in (EC.41b) holds for any subscriber set. Moreover, the total supply in the left-hand side is only determined by Q_s and independent from any parameter in the right-hand side. Hence, the market clearing price $\hat{p}_s$ can be controlled by varying Q_s. In particular, choose Q_s so that $\hat{p}_s = \hat{p}_n$. Similar to the derivations in Eqs. (EC.43) and (EC.44), we conclude that $\overline{\Pi}_n^0 = \overline{\Pi}_s \leq \overline{\Pi}_s$, where $\overline{\Pi}_s$ represents the revenue of the sharing contract(p_s, Q_s).

At last, we show $\Theta_s = \Theta_n^0$. Recall that we have proved $\overline{\Pi}_s = \overline{\Pi}_n^0 \leq \overline{\Pi}_n^0$ and $\Theta_s = \Theta_n^0$ for the nonlinear contract $(p_n, Q_n, \hat{p}_n) = (p_s - \hat{p}_s Q_s, 0, \hat{p}_s)$. Since $\overline{\Pi}_s = \overline{\Pi}_n^0$, then $\overline{\Pi}_n^0 = \overline{\Pi}_n^0$. Therefore, nonlinear contract $(p_n, Q_n, \hat{p}_n)$ is the optimal two-part tariff contract and has the same subscribers as the optimal sharing contract $(p_s,\ Q_s)$, i.e., $\Theta_s = \Theta_n^0$.

Proof of Theorem 4. First, we show Π_s is decreasing in w_u. For given w_u, the optimal sharing contract (p_s, Q_s), the corresponding equilibrium market clearing price $\hat{p}_s$, and the subscribing threshold $\bar{\theta}_s$ should satisfy

$$\begin{cases}
\dfrac{(\bar{\theta}_s - \hat{p}_s + w_u Q_s)^2}{2(1 + w_u)} - \dfrac{1}{2} w_u {Q_s}^2 + \hat{p}_s Q_s - p = 0, & \text{(EC.45)} \\[2ex]
\displaystyle\int_{\hat{p}_s + Q_s}^{\Theta} \frac{\theta - \hat{p}_s - Q_s}{1 + w_o} f(\theta) \mathrm{d}\theta = \int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} \frac{\hat{p}_s + Q_s - \theta}{1 + w_u} f(\theta) \mathrm{d}\theta, & \text{(EC.46)}
\end{cases}$$

where (EC.45) holds because the surplus of type-$\bar{\theta}_s$ customer is zero, (EC.46) holds because demand equals to supply in sharing market. We now show $\bar{\theta}_s$ is increasing in w_u for given (p_s, Q_s). We prove it by contradiction. Suppose $\bar{\theta}_s$ is decreasing in w_u for given (p_s, Q_s), i.e., $\frac{d\bar{\theta}_s}{dw_u} \leq 0$. Taking the derivative of both sides in (EC.45) and (EC.46) with respect to w_u, we get

$$\begin{cases} (\frac{\bar{\theta}_s - \hat{p}_s + w_u Q_s}{1+w_u} + Q_s)\frac{d\bar{\theta}_s}{dw_u} - \frac{(\bar{\theta}_s - \hat{p}_s - Q_s)^2}{(1+w_u)^2} = \frac{\bar{\theta}_s - \hat{p}_s - Q_s}{1+w_u}\frac{d\hat{p}_s}{dw_u}, & \text{(EC.47)} \\ -\int_{\hat{p}_s+Q_s}^{\Theta} \frac{1}{1+w_o} f(\theta)\mathrm{d}\theta \frac{d\hat{p}_s}{dw_u} = \int_{\bar{\theta}_s}^{\hat{p}_s+Q_s} \frac{(1+w_u)\frac{d\hat{p}_s}{dw_u} + \theta - \hat{p}_s - Q_s}{(1+w_u)^2} f(\theta)\mathrm{d}\theta. & \text{(EC.48)} \end{cases}$$

Recall $\hat{p}_s - w_u Q_s \leq \bar{\theta}_s < \hat{p}_s + Q_s$, together with $\frac{d\bar{\theta}_s}{dw_u} \leq 0$, (EC.47) implies $\frac{d\hat{p}_s}{dw_u} > 0$. Then the left side of (EC.48) is less than 0. Furthermore, (EC.47) is equivalent to $(\frac{\bar{\theta}_s - \hat{p}_s + w_u Q_s}{1+w_u} + Q_s)\frac{d\bar{\theta}_s}{dw_u} = \frac{(\bar{\theta}_s - \hat{p}_s - Q_s)}{(1+w_u)^2}[(1+w_u)\frac{d\hat{p}_s}{dw_u} + \bar{\theta}_s - \hat{p}_s - Q_s] > 0$. Thus, $\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s} \frac{(1+w_u)\frac{d\hat{p}_s}{dw_u} + \theta - \hat{p}_s - Q_s}{(1+w_u)^2} f(\theta)\mathrm{d}\theta \geq \int_{\bar{\theta}_s}^{\hat{p}_s+Q_s} \frac{(1+w_u)\frac{d\hat{p}_s}{dw_u} + \bar{\theta}_s - \hat{p}_s - Q_s}{(1+w_u)^2} f(\theta)\mathrm{d}\theta > 0$, i.e., the right side of (EC.48) is greater than 0. It is a contradiction. Hence, under the same sharing contract (p_s, Q_s), if we decrease w_u, more customer will subscribe (a smaller $\bar{\theta}_s$), which leads to higher revenue. Let $w_u = \beta w_o$, it is easy to see the sharing contract reduces to bucket contract if β is sufficiently large because all customers will use Q_s unit data when the underage rate $w_u = \beta w_o$ is large enough. In other words, there is no trade-in. Therefore, the nonlinear contract can yield higher revenue if β is sufficiently large. Next, we will show the sharing contract can yield higher revenue than nonlinear contract when $\beta = 1$. Thus, there exists a threshold $\bar{\beta} < 1$, the sharing contract yields higher revenue than nonlinear contract if and only if $w_u \leq \bar{\beta} w_o$ since the optimal revenue of nonlinear contract is independent on w_u. Let $\gamma = 1/\bar{\beta}$, we get the desired results in Theorem 4.

Now we show the sharing contract can yield higher revenue than nonlinear contract when $\beta = 1$, i.e., $w_u = w_o$. By (EC.99), (EC.101) and $Q_n = \bar{\theta}_n - \hat{p}_n$, the optimal revenue under nonlinear contract is

$$\Pi_n(p_n, Q_n, \hat{p}_n) = (\frac{1}{2}\bar{\theta}_n{}^2 - \bar{\theta}_n\hat{p}_n - \frac{1}{2}\hat{p}_n{}^2 + \frac{w_o\bar{\theta}_n\hat{p}_n}{1+w_o})\bar{F}(\bar{\theta}_n) + \hat{p}_n\int_{\bar{\theta}_n}^{\Theta} \frac{\theta}{1+w_o} f(\theta)\mathrm{d}\theta = (\frac{1}{2}\bar{\theta}_n{}^2 + \frac{1}{2}\hat{p}_n{}^2)\bar{F}(\bar{\theta}_n).$$

We construct a sharing contract (p_s, Q_s), where $p_s = \frac{(\int_{\bar{\theta}_n}^{\Theta} \theta f(\theta)\mathrm{d}\theta/\bar{F}(\bar{\theta}_n) - \bar{\theta}_n)^2}{2(1+w_o)} + \frac{\bar{\theta}_n^2}{2}$ and $Q_s = \bar{\theta}_n$. We will show $\bar{\theta}_s = \bar{\theta}_n$ and market clearing price $\hat{p}_s = \int_{\bar{\theta}_n}^{\Theta} \theta f(\theta)\mathrm{d}\theta/\bar{F}(\bar{\theta}_n) - \bar{\theta}_n$ by the following three steps.

First, we show $\frac{(\bar{\theta}_s - \hat{p}_s + w_o Q_s)^2}{2(1+w_o)} - \frac{1}{2}w_o Q_s^2 - p_s + \hat{p}_s Q_s = 0$, i.e., $s_s(d_s(\bar{\theta}_s)|\bar{\theta}_s) = 0$. That is

$$\begin{aligned} &\frac{(\bar{\theta}_s - \hat{p}_s + w_o Q_s)^2}{2(1+w_o)} - \frac{1}{2}w_o Q_s^2 - p_s + \hat{p}_s Q_s \\ =&\frac{1}{2}(1+w_o)(\bar{\theta}_n{}^2 - \hat{p}_n)^2 - \frac{1}{2}w_o\bar{\theta}_n{}^2 - p_s + (1+w_o)\hat{p}_n\bar{\theta}_n \\ =&\frac{1}{2}(1+w_o)(\bar{\theta}_n{}^2 - \hat{p}_n)^2 - \frac{1}{2}w_o\bar{\theta}_n{}^2 - \frac{1}{2}(1+w_o)\hat{p}_n{}^2 - \frac{1}{2}\bar{\theta}_n{}^2 + (1+w_o)\hat{p}_n\bar{\theta}_n = 0, \end{aligned}$$

where the first equality is due to $\hat{p}_s = (1+w_o)\hat{p}_n = \int_{\bar{\theta}_n}^{\Theta} \theta f(\theta)\mathrm{d}\theta/\bar{F}(\bar{\theta}_n) - \bar{\theta}_n$ and $\bar{\theta}_s = Q_s = \bar{\theta}_n$, the second equality is due to $p_s = (\int_{\bar{\theta}_n}^{\Theta} \theta f(\theta)\mathrm{d}\theta/\bar{F}(\bar{\theta}_n) - \bar{\theta}_n)^2/[2(1+w_o)] + \bar{\theta}_n^2\ /2 = (1+w_o)\hat{p}_n{}^2/2 + \bar{\theta}_n^2\ /2$. Second, we show (TS.19) holds for $Q_s = \bar{\theta}_s = \bar{\theta}_n$ and $\hat{p}_s = \int_{\bar{\theta}_n}^{\Theta} \theta f(\theta)\mathrm{d}\theta/\bar{F}(\bar{\theta}_n) - \bar{\theta}_n$. It is easy to see (TS.19) reduces to $Q_s\bar{F}(\bar{\theta}_s) = \int_{\bar{\theta}_s}^{\Theta}(\theta - \hat{p}_s)f(\theta)\mathrm{d}\theta$ when $w_u = w_o$. In other words, $\hat{p}_s = \int_{\bar{\theta}_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta/\bar{F}(\bar{\theta}_s) - Q_s$, which holds for $Q_s = \bar{\theta}_s = \bar{\theta}_n$ and $\hat{p}_s = \int_{\bar{\theta}_n}^{\Theta} \theta f(\theta)\mathrm{d}\theta/\bar{F}(\bar{\theta}_n) - \bar{\theta}_n$. Third, we show $\hat{p}_s \leq p_s/Q_s + w_u Q_s/2$ and $\bar{\theta}_s \geq \hat{p}_s - w_u Q_s$, then the equilibrium condition in Lemma TS7(ii) is satisfied with (TS.19) holds. We have $p_s/Q_s + w_u Q_s/2 - \hat{p}_s = (1+w_o)\hat{p}_n{}^2/(2\bar{\theta}_n) + \bar{\theta}_n/2 + w_o\bar{\theta}_n/2 - (1+w_o)\hat{p}_n = (1+w_o)(\hat{p}_n - \bar{\theta}_n)^2/(2\bar{\theta}_n) \geq 0$, where the first equality is due to $p_s = (1+w_o)\hat{p}_n{}^2/2 + \bar{\theta}_n^2\ /2$, $\hat{p}_s = (1+w_o)\hat{p}_n$, $Q_s = \bar{\theta}_n$

and $w_u = w_o$. Furthermore, $\bar{\theta}_s - (\hat{p}_s - w_u Q_s) = (1+w_o)(\bar{\theta}_n - \hat{p}_n) = (1+w_o)Q_n \geq 0$, where the first equality is due to $\hat{p}_s = (1+w_o)\hat{p}_n$, $Q_s = \bar{\theta}_s = \bar{\theta}_n$ and $w_u = w_o$, and the second equality is due to $Q_n = \bar{\theta}_n - \hat{p}_n$.

At last, we show $\Pi_s(p_s, Q_s) > (\frac{1}{2}\bar{\theta}_n{}^2 + \frac{1}{2}\hat{p}_n{}^2)\bar{F}(\bar{\theta}_n) = \Pi_n(p_n, Q_n, \hat{p}_n)$. That is,

$$\Pi_s(p_s, Q_s) = p_s \bar{F}(\bar{\theta}_s) = (\frac{1}{2}\bar{\theta}_n{}^2 + \frac{1}{2}(1+w_o)\hat{p}_n{}^2)\bar{F}(\bar{\theta}_n) > (\frac{1}{2}\bar{\theta}_n{}^2 + \frac{1}{2}\hat{p}_n{}^2)\bar{F}(\bar{\theta}_n)$$

where the second equality is due to $p_s = (1+w_o)\hat{p}_n{}^2/2 + \bar{\theta}_n^2\ /2$ and $\bar{\theta}_s = \bar{\theta}_n$, and the inequality is due to $w_o > 0$.

EC3. Other Proofs in Main Body

Proof of Proposition 1. According to Lemma TS1, customers' subscription decisions have a threshold structure; we thus write the provider's revenue-maximizing problem as

$$\max_{p_s \geq 0, Q_s \geq 0, t_s \geq 0} \Pi_s(p_s, Q_s, t_s) = p_s \cdot \bar{F}(\bar{\theta}_s(p_s, Q_s, t_s)) + t_s \cdot \int_{\bar{\theta}_s}^{\Theta} |d_s(\theta) - Q_s| f(\theta) d\theta \tag{EC.49}$$

where $\bar{\theta}_s(p_s, Q_s)$ is the cutoff for customer subscriptions as identified in Lemma TS1.

First, consider the case with speculators. By Lemma TS2(i), $\hat{p}_s \geq p_s/Q_s + t_s$. By (TS.1), we can rewrite (EC.49) as

$$\begin{aligned}
\Pi_s(p_s, Q_s, t_s) &= p_s \bar{F}(\bar{\theta}_s) + t_s \int_{\bar{\theta}_s}^{\hat{p}_s - t_s} Q_s f(\theta)\mathrm{d}\theta + t_s \int_{\hat{p}_s - t_s}^{\hat{p}_s + Q_s - t_s} (\hat{p}_s + Q_s - t_s - \theta) f(\theta)\mathrm{d}\theta \\
&\quad + t_s \int_{\hat{p}_s + Q_s + t_s}^{\Theta} (\theta - \hat{p}_s - Q_s - t_s) f(\theta)\mathrm{d}\theta \\
&= p_s \bar{F}(\bar{\theta}_s) + 2t_s \int_{\bar{\theta}_s}^{\hat{p}_s - t_s} Q_s f(\theta)\mathrm{d}\theta + 2t_s \int_{\hat{p}_s - t_s}^{\hat{p}_s + Q_s - t_s} (\hat{p}_s + Q_s - t_s - \theta) f(\theta)\mathrm{d}\theta \\
&\leq (\hat{p}_s - t_s) Q_s \bar{F}(\bar{\theta}_s) + 2t_s \int_{\bar{\theta}_s}^{\hat{p}_s - t_s} Q_s f(\theta)\mathrm{d}\theta + 2t_s \int_{\hat{p}_s - t_s}^{\hat{p}_s + Q_s - t_s} (\hat{p}_s + Q_s - t_s - \theta) f(\theta)\mathrm{d}\theta,
\end{aligned} \tag{EC.50}$$

where the second equality is from the equivalence of (TS.5) and (TS.6), and (EC.50) is due to $\hat{p}_s \geq p_s/Q_s + t_s$.

We next show that the inequality (EC.50) can hold with equality all the time, i.e., one can always choose (p_s, Q_s, t_s) such that $\hat{p}_s = p_s/Q_s + t_s$. Let $\underline{t}_s = 2t_s$ and $\underline{\hat{p}}_s = \hat{p}_s - t_s$. Then, (TS.3) is equivalent to

$$\int_{\bar{\theta}_s}^{\underline{\hat{p}}_s} Q_s f(\theta)\mathrm{d}\theta + \int_{\underline{\hat{p}}_s}^{\underline{\hat{p}}_s + Q_s} (\underline{\hat{p}}_s + Q_s - \theta) f(\theta)\mathrm{d}\theta = \int_{\underline{\hat{p}}_s + Q_s + \underline{t}_s}^{\Theta} (\theta - \underline{\hat{p}}_s - Q_s - \underline{t}_s) f(\theta)\mathrm{d}\theta. \tag{EC.51}$$

Note that for any given $(Q_s, \underline{t}_s)$ there exists a $\underline{\hat{p}}_s$ defined by (EC.51). Then, by setting $p_s = \underline{\hat{p}}_s Q_s$, we have $\hat{p}_s = p_s/Q_s + t_s$, which ascertains the equality of (EC.50). Thus, we can write $\Pi_s(p_s, Q_s, t_s)$ as

$$\Pi_s(\underline{\hat{p}}_s, Q_s, \underline{t}_s) = \underline{\hat{p}}_s Q_s \bar{F}(\bar{\theta}_s) + \underline{t}_s \int_{\bar{\theta}_s}^{\underline{\hat{p}}_s} Q_s f(\theta)\mathrm{d}\theta + \underline{t}_s \int_{\underline{\hat{p}}_s}^{\underline{\hat{p}}_s + Q_s} (\underline{\hat{p}}_s + Q_s - \theta) f(\theta)\mathrm{d}\theta, \tag{EC.52}$$

And we will show that the optimal profit must be achieved on the boundary.

Consider the first-order conditions of $\Pi_s(\underline{\hat{p}}_s, Q_s, \underline{t}_s)$ in (EC.52)

$$\begin{cases} \dfrac{\partial \Pi_s}{\partial \hat{p}_s} = -(\hat{p}_s + t_s) Q_s f(\bar{\theta}_s) \dfrac{\partial \bar{\theta}_s}{\partial \hat{p}_s} + Q_s \bar{F}(\bar{\theta}_s) + t_s \displaystyle\int_{\hat{p}_s}^{\hat{p}_s + Q_s} f(\theta) \mathrm{d}\theta = 0 & \text{(EC.53)} \\ \dfrac{\partial \Pi_s}{\partial Q_s} = -(\hat{p}_s + t_s) Q_s f(\bar{\theta}_s) \dfrac{\partial \bar{\theta}_s}{\partial Q_s} + \hat{p}_s \bar{F}(\bar{\theta}_s) + t_s \displaystyle\int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} f(\theta) \mathrm{d}\theta = 0 & \text{(EC.54)} \\ \dfrac{\partial \Pi_s}{\partial t_s} = -(\hat{p}_s + t_s) Q_s f(\bar{\theta}_s) \dfrac{\partial \bar{\theta}_s}{\partial t_s} + \displaystyle\int_{\hat{p}_s + Q_s + t_s}^{\Theta} (\theta - \hat{p}_s - Q_s - t_s) f(\theta) \mathrm{d}\theta = 0, & \text{(EC.55)} \end{cases}$$

which connect the optimal contact with the subscribing threshold $\bar{\theta}_s$.

To establish the result, we need an intermediate step of proving

$$\frac{Q_s \bar{F}(\bar{\theta}_s)}{\frac{\partial \bar{\theta}_s}{\hat{p}_s}} \geq \frac{\int_{\hat{p}_s + Q_s + t_s}^{\Theta} (\theta - \hat{p}_s - Q_s - t_s) f(\theta) \mathrm{d}\theta}{\frac{\partial \bar{\theta}_s}{\partial t_s}}, \tag{EC.56}$$

or equivalently,

$$\frac{Q_s \bar{F}(\bar{\theta}_s)}{\int_{\hat{p}_s + Q_s + t_s}^{\Theta} (\theta - \hat{p}_s - Q_s - t_s) f(\theta) \mathrm{d}\theta} \geq \frac{\partial \bar{\theta}_s}{\partial \hat{p}_s} \Big/ \frac{\partial \bar{\theta}_s}{\partial t_s}. \tag{EC.57}$$

First, let us change the form of the left-hand side of (EC.57). Consider the right-hand side of (EC.51)

$$\begin{aligned} \int_{\hat{p}_s + Q_s + t_s}^{\Theta} (\theta - \hat{p}_s - Q_s - t_s) f(\theta) \mathrm{d}\theta &= -\int_{\hat{p}_s + Q_s + t_s}^{\Theta} (\theta - \hat{p}_s - Q_s - t_s) \mathrm{d}\bar{F}(\theta) \\ &= -(\theta - \hat{p}_s - Q_s - t_s) \bar{F}(\theta) \big|_{\hat{p}_s + Q_s + t_s}^{\Theta} + \int_{\hat{p}_s + Q_s + t_s}^{\Theta} \bar{F}(\theta) \mathrm{d}\theta \\ &= \int_{\hat{p}_s + Q_s + t_s}^{\Theta} \bar{F}(\theta) \mathrm{d}\theta \end{aligned} \tag{EC.58}$$

and the left-hand side of (EC.51)

$$\begin{aligned} \int_{\bar{\theta}_s}^{\hat{p}_s} Q_s f(\theta) \mathrm{d}\theta + \int_{\hat{p}_s}^{\hat{p}_s + Q_s} (\hat{p}_s + Q_s - \theta) f(\theta) \mathrm{d}\theta &= \int_{\bar{\theta}_s}^{\hat{p}_s} Q_s f(\theta) \mathrm{d}\theta - \int_{\hat{p}_s}^{\hat{p}_s + Q_s} (\hat{p}_s + Q_s - \theta) \mathrm{d}\bar{F}(\theta) \\ &= \int_{\bar{\theta}_s}^{\hat{p}_s} Q_s f(\theta) \mathrm{d}\theta - (\hat{p}_s + Q_s - \theta) \bar{F}(\theta) \Big|_{\hat{p}_s}^{\hat{p}_s + Q_s} - \int_{\hat{p}_s}^{\hat{p}_s + Q_s} \bar{F}(\theta) \mathrm{d}\theta \\ &= Q_s \bar{F}(\bar{\theta}_s) - \int_{\hat{p}_s}^{\hat{p}_s + Q_s} \bar{F}(\theta) \mathrm{d}\theta. \qquad \text{(EC.59)} \\ \text{by (EC.51, EC.58)} \quad &= \int_{\hat{p}_s + Q_s + t_s}^{\Theta} \bar{F}(\theta) \mathrm{d}\theta, \qquad \text{(EC.60)} \end{aligned}$$

respectively. Combining (EC.58), (EC.59), and (EC.60), we have

$$\frac{Q_s \bar{F}(\bar{\theta}_s)}{\int_{\hat{p}_s + Q_s + t_s}^{\Theta} (\theta - \hat{p}_s - Q_s - t_s) f(\theta) \mathrm{d}\theta} = \frac{\int_{\hat{p}_s + Q_s + t_s}^{\Theta} \bar{F}(\theta) \mathrm{d}\theta + \int_{\hat{p}_s}^{\hat{p}_s + Q_s} \bar{F}(\theta) \mathrm{d}\theta}{\int_{\hat{p}_s + Q_s + t_s}^{\Theta} \bar{F}(\theta) \mathrm{d}\theta} = 1 + \frac{\int_{\hat{p}_s}^{\hat{p}_s + Q_s} \bar{F}(\theta) \mathrm{d}\theta}{\int_{\hat{p}_s + Q_s + t_s}^{\Theta} \bar{F}(\theta) \mathrm{d}\theta}. \tag{EC.61}$$

Next, let us change the form of the right-hand side of (EC.57). Taking the first derivatives of both sides of (EC.51) with respect to $\hat{p}_s$, Q_s, t_s respectively, we have

$$\begin{cases} Q_s f(\bar{\theta}_s) \dfrac{\partial \bar{\theta}_s}{\partial \hat{p}_s} = \displaystyle\int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} f(\theta) \mathrm{d}\theta + \int_{\hat{p}_s + Q_s + t_s}^{\Theta} f(\theta) \mathrm{d}\theta, & \text{(EC.62)} \\ Q_s f(\bar{\theta}_s) \dfrac{\partial \bar{\theta}_s}{\partial Q_s} = \displaystyle\int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} f(\theta) \mathrm{d}\theta + \int_{\hat{p}_s + Q_s + t_s}^{\Theta} f(\theta) \mathrm{d}\theta + \int_{\bar{\theta}_s}^{\hat{p}_s} f(\theta) \mathrm{d}\theta, & \text{(EC.63)} \\ Q_s f(\bar{\theta}_s) \dfrac{\partial \bar{\theta}_s}{\partial t_s} = \displaystyle\int_{\hat{p}_s + Q_s + t_s}^{\Theta} f(\theta) \mathrm{d}\theta. & \text{(EC.64)} \end{cases}$$

Eqs. (EC.62) and (EC.64) indicate that

$$\frac{\partial \bar{\theta}_s}{\partial \hat{p}_s} \Big/ \frac{\partial \bar{\theta}_s}{\partial t_s} = \frac{\int_{\hat{p}_s}^{\hat{p}_s + Q_s} f(\theta) \mathrm{d}\theta + \int_{\hat{p}_s + Q_s + t_s}^{\Theta} f(\theta) \mathrm{d}\theta}{\int_{\hat{p}_s + Q_s + t_s}^{\Theta} f(\theta) \mathrm{d}\theta} = 1 + \frac{\int_{\hat{p}_s}^{\hat{p}_s + Q_s} f(\theta) \mathrm{d}\theta}{\int_{\hat{p}_s + Q_s + t_s}^{\Theta} f(\theta) \mathrm{d}\theta} \tag{EC.65}$$

Combining (EC.61) and (EC.65), we have that

$$\frac{Q_s\bar{F}(\bar{\theta}_s)}{\int_{\hat{p}_s+Q_s+t_s}^{\Theta}(\theta-\hat{p}_s-Q_s-t_s)f(\theta)\mathrm{d}\theta} \geq \frac{\partial\bar{\theta}_s}{\partial\hat{p}_s}\Big/\frac{\partial\bar{\theta}_s}{\partial t_s} \qquad \frac{\int_{\hat{p}_s}^{\hat{p}_s+Q_s}\bar{F}(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s+Q_s+t_s}^{\Theta}\bar{F}(\theta)\mathrm{d}\theta} \geq \frac{\int_{\hat{p}_s}^{\hat{p}_s+Q_s}f(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s+Q_s+t_s}^{\Theta}f(\theta)\mathrm{d}\theta}$$

$$\frac{\int_{\hat{p}_s+Q_s+t_s}^{\Theta}f(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s+Q_s+t_s}^{\Theta}\bar{F}(\theta)\mathrm{d}\theta} \geq \frac{\int_{\hat{p}_s}^{\hat{p}_s+Q_s}f(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s}^{\hat{p}_s+Q_s}\bar{F}(\theta)\mathrm{d}\theta},$$

which holds because

$$\frac{\int_{\hat{p}_s+Q_s+t_s}^{\Theta}f(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s+Q_s+t_s}^{\Theta}\bar{F}(\theta)\mathrm{d}\theta} \geq \frac{\int_{\hat{p}_s+Q_s}^{\Theta}f(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s+Q_s}^{\Theta}\bar{F}(\theta)\mathrm{d}\theta} \geq \frac{\int_{\hat{p}_s}^{\Theta}f(\theta)\mathrm{d}\theta-\int_{\hat{p}_s+Q_s}^{\Theta}f(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s}^{\Theta}\bar{F}(\theta)\mathrm{d}\theta-\int_{\hat{p}_s+Q_s}^{\Theta}\bar{F}(\theta)\mathrm{d}\theta} = \frac{\int_{\hat{p}_s}^{\hat{p}_s+Q_s}f(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s}^{\hat{p}_s+Q_s}\bar{F}(\theta)\mathrm{d}\theta},$$

where the first inequality is directly from Assumption 1 and the second inequality results from $\frac{\int_{\hat{p}_s+Q_s}^{\Theta}f(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s+Q_s}^{\Theta}\bar{F}(\theta)\mathrm{d}\theta} \geq \frac{\int_{\hat{p}_s}^{\Theta}f(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s}^{\Theta}\bar{F}(\theta)\mathrm{d}\theta}$ by Assumption 1. Therefore, (EC.57) holds and so does (EC.56).

Now we are ready to show that there is no interior solutions to the first-order conditions by contradiction. Assume the optimal solution is determined by the first order conditions. Then, (EC.53) and (EC.55) indicate that

$$\frac{Q_s\bar{F}(\bar{\theta}_s)+t_s\int_{\hat{p}_s}^{\hat{p}_s+Q_s}f(\theta)\mathrm{d}\theta}{\frac{\partial\bar{\theta}_s}{\partial\hat{p}_s}} = \frac{\int_{\hat{p}_s+Q_s+t_s}^{\Theta}(\theta-\hat{p}_s-Q_s-t_s)f(\theta)\mathrm{d}\theta}{\frac{\partial\bar{\theta}_s}{\partial t_s}},$$

which together with (EC.56) implies that $t_s=0$ for any possible solution to the first-order conditions.

Note that it is also possible that there is no solution of the first-order conditions. In that case, optimal solutions may reside on the boundary such that at least one of $\hat{p}_s=0$, $Q_s=0$ and $t_s=0$ is true. We prove that only $t_s=0$ can hold. If $\hat{p}_s=0$, then reselling would not be profitable at all. If $Q_s=0$, then no sharing would exist since there are no supplies at all. Thus, it can only be $t_s=0$, or equivalently, $t_s=0$.

Second, consider the case with no speculators. By Lemma TS2(ii), $\bar{\theta}_s \geq \hat{p}_s - t_s$. By (TS.1) and (TS.2), we can rewrite (EC.49) as

$$\begin{aligned}\Pi_s(p_s,Q_s,t_s)=&\left[\frac{1}{2}(\theta-\hat{p}_s+t_s)^2+(\hat{p}_s-t_s)Q_s\right]\bar{F}(\bar{\theta}_s)\\&+t_s\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s-t_s}(\hat{p}_s+Q_s-t_s-\theta)f(\theta)\mathrm{d}\theta+t_s\int_{\hat{p}_s+Q_s+t_s}^{\Theta}(\theta-\hat{p}_s-Q_s-t_s)f(\theta)\mathrm{d}\theta\\=&\left[\frac{1}{2}(\theta-\hat{p}_s+t_s)^2+(\hat{p}_s-t_s)Q_s\right]\bar{F}(\bar{\theta}_s)+2t_s\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s-t_s}(\hat{p}_s+Q_s-t_s-\theta)f(\theta)\mathrm{d}\theta,\end{aligned} \tag{EC.66}$$

where the last equality is due to the equivalence of (TS.7) and (TS.8). Let $t_s=2t_s$ and $\hat{p}_s=\hat{p}_s-t_s$, thus, we can write $\Pi_s(p_s,Q_s,t_s)$ as

$$\Pi_s(\hat{p}_s,Q_s,t_s)=\left[\frac{1}{2}(\theta-\hat{p}_s)^2+\hat{p}_sQ_s\right]\bar{F}(\bar{\theta}_s)+t_s\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s}(\hat{p}_s+Q_s-\theta)f(\theta)\mathrm{d}\theta. \tag{EC.67}$$

Moreover, (TS.4) is equivalent to

$$\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s}(\hat{p}_s+Q_s-\theta)f(\theta)\mathrm{d}\theta=\int_{\hat{p}_s+Q_s+t_s}^{\Theta}(\theta-\hat{p}_s-Q_s-t_s)f(\theta)\mathrm{d}\theta. \tag{EC.68}$$

And we will show that the optimal profit must be achieved on the boundary.

Consider the first-order conditions of $\Pi_s(\hat{p}_s, Q_s, t_s)$ in (EC.67),

$$\begin{aligned}\frac{\partial \Pi_s}{\partial \hat{p}_s} = & \Big[(\bar{\theta}_s - \hat{p}_s)\frac{\bar{F}(\bar{\theta}_s)}{f(\bar{\theta}_s)} - (\frac{1}{2}(\bar{\theta}_s - \hat{p}_s)^2 + \hat{p}_s Q_s) - (\hat{p}_s + Q_s - \bar{\theta}_s) f\ \Big] f(\bar{\theta}_s)\frac{\partial \bar{\theta}_s}{\partial \hat{p}_s} \\ & + (-\bar{\theta}_s + \hat{p}_s + Q_s)\bar{F}(\bar{\theta}_s) + t_s \int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} f(\theta)\mathrm{d}\theta = 0,\end{aligned} \tag{EC.69}$$

$$\begin{aligned}\frac{\partial \Pi_s}{\partial Q_s} = & \Big[(\bar{\theta}_s - \hat{p}_s)\frac{\bar{F}(\bar{\theta}_s)}{f(\bar{\theta}_s)} - (\frac{1}{2}(\bar{\theta}_s - \hat{p}_s)^2 + \hat{p}_s Q_s) - (\hat{p}_s + Q_s - \bar{\theta}_s) t_s\Big] f(\bar{\theta}_s)\frac{\partial \bar{\theta}_s}{\partial Q_s} \\ & + \hat{p}_s \bar{F}(\bar{\theta}_s) + t_s \int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} f(\theta)\mathrm{d}\theta = 0,\end{aligned} \tag{EC.70}$$

$$\begin{aligned}\frac{\partial \Pi_s}{\partial t_s} = & \Big[(\bar{\theta}_s - \hat{p}_s)\frac{\bar{F}(\bar{\theta}_s)}{f(\bar{\theta}_s)} - (\frac{1}{2}(\bar{\theta}_s - \hat{p}_s)^2 + \hat{p}_s Q_s) - (\hat{p}_s + Q_s - \bar{\theta}_s) t_s\Big] f(\bar{\theta}_s)\frac{\partial \bar{\theta}_s}{\partial t_s} \\ & + \int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} (\hat{p}_s + Q_s - \theta) f(\theta)\mathrm{d}\theta = 0,\end{aligned} \tag{EC.71}$$

which connect the optimal contact with the subscribing threshold $\bar{\theta}_s$.

To establish the result, we need an intermediate step of proving

$$\frac{\hat{p}_s \bar{F}(\bar{\theta}_s)}{\frac{\partial \bar{\theta}_s}{\partial Q_s}} \geq \frac{\int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} (\hat{p}_s + Q_s - \theta) f(\theta)\mathrm{d}\theta}{\frac{\partial \bar{\theta}_s}{\partial t_s}}, \tag{EC.72}$$

or equivalently,

$$\frac{\hat{p}_s \bar{F}(\bar{\theta}_s)}{\int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} (\hat{p}_s + Q_s - \theta) f(\theta)\mathrm{d}\theta} \geq \frac{\partial \bar{\theta}_s}{\partial Q_s} \Big/ \frac{\partial \bar{\theta}_s}{\partial t_s}. \tag{EC.73}$$

First, let us change the form of the left-hand side of (EC.73). One one hand,

$$\begin{aligned}\int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} (\hat{p}_s + Q_s - \theta) f(\theta)\mathrm{d}\theta = & -\int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} (\hat{p}_s + Q_s - \theta)\mathrm{d}\bar{F}(\theta) \\ = & -(\hat{p}_s + Q_s - \theta)\bar{F}(\theta)\big|_{\bar{\theta}_s}^{\hat{p}_s + Q_s} - \int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} \bar{F}(\theta)\mathrm{d}\theta \\ = & \hat{p}_s \bar{F}(\bar{\theta}_s) - \int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} \bar{F}(\theta)\mathrm{d}\theta.\end{aligned} \tag{EC.74}$$

On the other hand,

$$\begin{aligned}\int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} (\hat{p}_s + Q_s - \theta) f(\theta)\mathrm{d}\theta \overset{(EC.68)}{=} & \int_{\hat{p}_s + Q_s + t_s}^{\Theta} (\theta - \hat{p}_s - Q_s - t_s) f(\theta)\mathrm{d}\theta \\ = & -\int_{\hat{p}_s + Q_s + t_s}^{\Theta} (\theta - \hat{p}_s - Q_s - t_s)\mathrm{d}\bar{F}(\theta) \\ = & -(\theta - \hat{p}_s - Q_s - t_s)\bar{F}(\theta)\big|_{\hat{p}_s + Q_s + t_s}^{\Theta} + \int_{\hat{p}_s + Q_s + t_s}^{\Theta} \bar{F}(\theta)\mathrm{d}\theta \\ = & \int_{\hat{p}_s + Q_s + t_s}^{\Theta} \bar{F}(\theta)\mathrm{d}\theta.\end{aligned} \tag{EC.75}$$

Combining (EC.74), (EC.75), we have

$$\frac{\hat{p}_s \bar{F}(\bar{\theta}_s)}{\int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} (\hat{p}_s + Q_s - \theta) f(\theta)\mathrm{d}\theta} = \frac{\int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} \bar{F}(\theta)\mathrm{d}\theta + \int_{\hat{p}_s + Q_s + t_s}^{\Theta} \bar{F}(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s + Q_s + t_s}^{\Theta} \bar{F}(\theta)\mathrm{d}\theta} = 1 + \frac{\int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} \bar{F}(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s + Q_s + t_s}^{\Theta} \bar{F}(\theta)\mathrm{d}\theta}. \tag{EC.76}$$

Next, let us change the form of the right-hand side of (EC.73). Taking the first derivatives of both sides of (EC.68) with respect to $\hat{p}_s$, Q_s, t_s respectively, we have

$$\begin{cases} (\hat{p}_s + Q_s - \bar{\theta}_s) f(\bar{\theta}_s) \dfrac{\partial \bar{\theta}_s}{\partial \hat{p}_s} = \displaystyle\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s} f(\theta)\mathrm{d}\theta + \int_{\hat{p}_s+Q_s+t_s}^{\Theta} f(\theta)\mathrm{d}\theta, & \text{(EC.77)} \\ (\hat{p}_s + Q_s - \bar{\theta}_s) f(\bar{\theta}_s) \dfrac{\partial \bar{\theta}_s}{\partial Q_s} = \displaystyle\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s} f(\theta)\mathrm{d}\theta + \int_{\hat{p}_s+Q_s+t_s}^{\Theta} f(\theta)\mathrm{d}\theta, & \text{(EC.78)} \\ (\hat{p}_s + Q_s - \bar{\theta}_s) f(\bar{\theta}_s) \dfrac{\partial \bar{\theta}_s}{\partial t_s} = \displaystyle\int_{\hat{p}_s+Q_s+t_s}^{\Theta} f(\theta)\mathrm{d}\theta. & \text{(EC.79)} \end{cases}$$

Eqs. (EC.78) and (EC.79) indicate that

$$\frac{\partial \bar{\theta}_s}{\partial Q_s} \Big/ \frac{\partial \bar{\theta}_s}{\partial t_s} = \frac{\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s} f(\theta)\mathrm{d}\theta + \int_{\hat{p}_s+Q_s+t_s}^{\Theta} f(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s+Q_s+t_s}^{\Theta} f(\theta)\mathrm{d}\theta} = 1 + \frac{\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s} f(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s+Q_s+t_s}^{\Theta} f(\theta)\mathrm{d}\theta}. \tag{EC.80}$$

Combining (EC.76) and (EC.80), we have that

$$\frac{\hat{p}_s \bar{F}(\bar{\theta}_s)}{\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s} (\hat{p}_s + Q_s - \theta) f(\theta)\mathrm{d}\theta} \geq \frac{\partial \bar{\theta}_s}{\partial Q_s} \Big/ \frac{\partial \bar{\theta}_s}{\partial t_s} \qquad \frac{\int_{\hat{p}_s}^{\hat{p}_s+Q_s} \bar{F}(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s+Q_s+t_s}^{\Theta} \bar{F}(\theta)\mathrm{d}\theta} \geq \frac{\int_{\hat{p}_s}^{\hat{p}_s+Q_s} f(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s+Q_s+t_s}^{\Theta} f(\theta)\mathrm{d}\theta}$$

$$\frac{\int_{\hat{p}_s+Q_s+t_s}^{\Theta} f(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s+Q_s+t_s}^{\Theta} \bar{F}(\theta)\mathrm{d}\theta} \geq \frac{\int_{\hat{p}_s}^{\hat{p}_s+Q_s} f(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s}^{\hat{p}_s+Q_s} \bar{F}(\theta)\mathrm{d}\theta},$$

which holds because

$$\frac{\int_{\hat{p}_s+Q_s+t_s}^{\Theta} f(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s+Q_s+t_s}^{\Theta} \bar{F}(\theta)\mathrm{d}\theta} \geq \frac{\int_{\hat{p}_s+Q_s}^{\Theta} f(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s+Q_s}^{\Theta} \bar{F}(\theta)\mathrm{d}\theta} \geq \frac{\int_{\hat{p}_s}^{\Theta} f(\theta)\mathrm{d}\theta - \int_{\hat{p}_s+Q_s}^{\Theta} f(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s}^{\Theta} \bar{F}(\theta)\mathrm{d}\theta - \int_{\hat{p}_s+Q_s}^{\Theta} \bar{F}(\theta)\mathrm{d}\theta} = \frac{\int_{\hat{p}_s}^{\hat{p}_s+Q_s} f(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s}^{\hat{p}_s+Q_s} \bar{F}(\theta)\mathrm{d}\theta},$$

where the first inequality is directly from Assumption 1 and the second inequality results from $\frac{\int_{\hat{p}_s+Q_s}^{\Theta} f(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s+Q_s}^{\Theta} \bar{F}(\theta)\mathrm{d}\theta} \geq \frac{\int_{\hat{p}_s}^{\Theta} f(\theta)\mathrm{d}\theta}{\int_{\hat{p}_s}^{\Theta} \bar{F}(\theta)\mathrm{d}\theta}$ by Assumption 1. Therefore, (EC.73) holds and so does (EC.72).

Now we are ready to show that there is no interior solutions to the first-order conditions by contradiction. Assume the optimal solution is determined by the first order conditions. Then, (EC.70) and (EC.71) indicate that

$$\frac{\hat{p}_s \bar{F}(\bar{\theta}_s) + t_s \int_{\bar{\theta}_s}^{\hat{p}_s+Q_s} f(\theta)\mathrm{d}\theta}{\frac{\partial \bar{\theta}_s}{\partial Q_s}} = \frac{\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s} (\hat{p}_s + Q_s - \theta) f(\theta)\mathrm{d}\theta}{\frac{\partial \bar{\theta}_s}{\partial t_s}},$$

which together with (EC.72) implies that $t_s = 0$ for any possible solution to the first-order conditions.

Note that it is also possible that there is no solution of the first-order conditions. In that case, optimal solutions may reside on the boundary such that at least one of $\hat{p}_s = 0$, $Q_s = 0$ and $t_s = 0$ is true. We prove that only $t_s = 0$ can hold. If $\hat{p}_s = 0$, then reselling would not be profitable at all. If $Q_s = 0$, then no sharing would exist since there are no supplies at all. Thus, it can only be $t_s = 0$, or equivalently, $t_s = 0$.

Proof of Lemma 1. If a customer decides to subscribe to the service, i.e., $s_s(d_s \mid \theta) \geq 0$, we can derive her marginal utility change $\frac{\partial s_s}{\partial d_s} = \theta - d_s - \hat{p}_s$. First, there must be a unique optimal d_s because the marginal utility change is monotone in d_s. Moreover, since $d_s \geq 0$, we have that if $\theta < \hat{p}_s$, then $\frac{\partial s_s}{\partial d_s} < 0$ and $d_s = 0$; Otherwise, $s_s(d_s \mid \theta)$ is maximized at $d_s(\theta) = \theta - \hat{p}_s$. By (4),

$$d_s(\theta) = \begin{cases} \theta - \hat{p}_s, & \text{if } \theta \geq \hat{p}_s \\ 0, & \text{otherwise} \end{cases} \text{ and } s_s(d_s(\theta) \mid \theta) = \begin{cases} \frac{1}{2}(\theta - \hat{p}_s)^2 + \hat{p}_s Q_s - p_s, & \text{if } \theta \geq \hat{p}_s \\ \hat{p}_s Q_s - p_s, & \text{otherwise,} \end{cases}$$

which shows that $s_s(d_s(\theta) \mid \theta)$ strictly increases in θ. Hence, there exists a unique $\bar{\theta}_s \geq 0$ such that $s_s(d_s(\theta) \mid \theta) \geq 0$ if and only if $\theta \geq \bar{\theta}_s$. In particular, let $\bar{\theta}_s \geq 0$ be the solution to

$$s_s\left(d_s(\theta)|\theta\right) = \frac{1}{2}\left(\theta - \hat{p}_s\right)^2 + \hat{p}_s Q_s - p_s = 0. \tag{EC.81}$$

We claim that customers subscribe to the service if and only if $\theta \geq \bar{\theta}_s$.

Proof of Lemma 2. (i) By Lemma 1, $d_s(\theta) = \max\{\theta - \hat{p}_s, 0\}$. For speculators who does not consume any data, $d_s(\theta) = \max\{\theta - \hat{p}_s, 0\} = 0$, which occurs if and only if $\bar{\theta}_s < \hat{p}_s$. For these speculators, the fact that they subscribe to the service indicates that $s_s(d_s(\theta) \mid \theta) = \hat{p}_s Q_s - p_s \geq 0$. Thus, $\hat{p}_s \geq p_s/Q_s$. In this case, the total supply and demand of the sharing market are

$$\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s} [Q_s - d_s(\theta)] f(\theta)\mathrm{d}\theta = \int_{\bar{\theta}_s}^{\hat{p}_s} Q_s f(\theta)\mathrm{d}\theta + \int_{\hat{p}_s}^{\hat{p}_s+Q_s} \left(Q_s - (\theta - \hat{p}_s)\right) f(\theta)\mathrm{d}\theta \tag{EC.82}$$

and

$$\int_{\hat{p}_s+Q_s}^{\Theta} \left(d_s(\theta) - Q_s\right) f(\theta)\mathrm{d}\theta = \int_{\hat{p}_s+Q_s}^{\Theta} \left((\theta - \hat{p}_s) - Q_s\right) f(\theta)\mathrm{d}\theta, \tag{EC.83}$$

respectively. Equating (EC.82) and (EC.83) to attain the market clearing condition, we obtain (5).

(ii) By Lemma 1, $d_s(\theta) = \max\{\theta - \hat{p}_s, 0\}$. The fact that there are no speculators means that $d_s(\theta) = \theta - \hat{p}_s$ for all subscribers, which occurs if and only if $\bar{\theta}_s \geq \hat{p}_s$. For these subscribers, $s_s(d_s(\theta) \mid \theta) \geq 0$. In particular, for the subscriber of type $\bar{\theta}_s$,

$$s_s(d_s(\bar{\theta}_s) \mid \bar{\theta}_s) = \frac{1}{2}(\bar{\theta}_s - \hat{p}_s)^2 - p_s + \hat{p}_s Q_s = 0 \text{ or } \text{ equivalently } \frac{1}{2}(\bar{\theta}_s - \hat{p}_s)^2 = p_s - \hat{p}_s Q_s \geq 0.$$

Thus, $\hat{p}_s \leq p_s/Q_s$. In this case, the total supply and demand of the sharing market are

$$\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s} \left(Q_s - d_s(\theta)\right) f(\theta)\mathrm{d}\theta = \int_{\bar{\theta}_s}^{\hat{p}_s+Q_s} \left(Q_s - (\theta - \hat{p}_s)\right) f(\theta)\mathrm{d}\theta \tag{EC.84}$$

and

$$\int_{\hat{p}_s+Q_s}^{\Theta} \left(d_s(\theta) - Q_s\right) f(\theta)\mathrm{d}\theta = \int_{\hat{p}_s+Q_s}^{\Theta} \left((\theta - \hat{p}_s) - Q_s\right) f(\theta)\mathrm{d}\theta, \tag{EC.85}$$

respectively. Equating (EC.84) and (EC.85) to attain the market clearing condition, we obtain (6).

Proof of Corollary 1. By Lemma 1, the demand of subscribers of type θ is $d_s(\theta) = \max\{\theta - \hat{p}_s, 0\}$. Therefore, subscribers will buy $\theta - \hat{p}_s - Q_s$ units in the sharing market if and only if $d_s(\theta) = \max\{\theta - \hat{p}_s, 0\} > Q_s$, i.e., $\theta > Q_s + \hat{p}_s$. By Lemma 2, customers subscribe to the service if and only if $\theta \geq \bar{\theta}_s$, thus subscribers of type $\theta \quad (\bar{\theta}_s, Q_s + \hat{p}_s)$ will sell data in the sharing market.

Proof of Lemma 3. We can write $s_n(d_n \mid \theta)$ in (9) as

$$s_n(d_n \mid \theta) = \begin{cases} -\frac{1}{2}d_n^2 + \theta d_n - p_n, & \text{if } d_n < Q_n \\ -\frac{1}{2}d_n^2 + (\theta - \hat{p}_n)d_n + \hat{p}_n Q_n - p_n, & \text{if } d_n \geq Q_n. \end{cases}$$

Note that $s_n(d_n \mid \theta)$ is concave in d_n when $d_n < Q_n$ and $d_n \geq Q_n$, respectively. Then, conditional on the fact that a customer has already subscribed to the service, i.e., $s_n(d_n \mid \theta) \geq 0$, we can derive her demand by the first order condition (FOC). Hence,

$$\frac{\partial s_n}{\partial d_n} = \begin{cases} \theta - d_n = 0, & \text{if } d_n < Q_n \quad \text{(EC.86)} \\ \theta - d_n - \hat{p}_n = 0, & \text{if } d_n \geq Q_n \quad \text{(EC.87)} \end{cases} \qquad d_n = \begin{cases} \theta, & \text{if } d_n < Q_n \quad \text{(EC.88)} \\ \theta - \hat{p}_n, & \text{if } d_n \geq Q_n. \quad \text{(EC.89)} \end{cases}$$

We next write d_n as function of customer type θ. First, if $\theta < Q_n$, it is easy to see that $d_n(\theta) = \theta$ by (EC.88). Second, if $\theta \geq \hat{p}_n + Q_n$, then by (EC.89) we have $d_n(\theta) = \theta - \hat{p}_n \geq Q_n$. At last, we show that $d_n(\theta) = Q_n$ if $Q_n \leq \theta < \hat{p}_n + Q_n$:

(i) Assume a type-θ customer consumes $d_n < Q_n$. By (EC.86), $\frac{\partial s_n}{\partial d_n} = \theta - d_n \geq Q_n - d_n > 0$, then the customer prefers increasing her demand to Q_n, i.e., $d_n(\theta) = Q_n$.

(ii) Assume a type-θ customer consumes $d_n \geq Q_n$. By (EC.87), $\frac{\partial s_n}{\partial d_n} = \theta - d_n - \hat{p}_n < \hat{p}_n + Q_n - d_n - \hat{p}_n = Q_n - d_n \leq 0$, then the customer prefers decreasing her demand to Q_n, i.e., $d_n(\theta) = Q_n$.

Proof of Lemma 4. By Lemma 3, we can write $s_n(d_n \mid \theta)$ in (9) as

$$s_n(d_n(\theta) \mid \theta) = \begin{cases} \frac{1}{2}\theta^2 - p_n, & \text{if } 0 \leq \theta < Q_n \quad \text{(EC.90)} \\ \theta Q_n - \frac{1}{2}Q_n^2 - p_n, & \text{if } Q_n \leq \theta < \hat{p}_n + Q_n \quad \text{(EC.91)} \\ \frac{1}{2}(\theta - \hat{p}_n)^2 - (p_n - \hat{p}_n Q_n), & \text{if } \hat{p}_n + Q_n \leq \theta \leq \Theta. \quad \text{(EC.92)} \end{cases}$$

If $p_n > \frac{1}{2}(\Theta - \hat{p}_n)^2 + \hat{p}_n Q_n$, $s_n(d_n(\theta = \Theta) \mid \theta = \Theta) < 0$. Since $s_n(d_n(\theta) \mid \theta)$ strictly increases in θ, no customers earn positive surplus and hence there are no subscribers. On the contrary, if $0 \leq p_n \leq \frac{1}{2}(\Theta - \hat{p}_n)^2 + \hat{p}_n Q_n$, $s_n(d_n(\theta = 0) \mid \theta = 0) \leq 0$ and $s_n(d_n(\theta = \Theta) \mid \theta = \Theta) \geq 0$. Therefore, there exists a unique $\bar{\theta}_n$ such that $s_n(d_n(\theta) \mid \theta) = 0$ and customers subscribe to the service if and only if $\theta \geq \bar{\theta}_n$.

Next, we characterize $\bar{\theta}_n$. First, let us consider the case $0 \leq p_n < Q_n^2/2$. For any customer of type $\theta \quad [Q_n, \hat{p}_n + Q_n)$, $s_n(d_n(\theta) \mid \theta) = \theta Q_n - \frac{1}{2}Q_n^2 - p_n > \theta Q_n - Q_n^2 \geq 0$ by eq. (EC.91). For any customer of type $\theta \quad [\hat{p}_n + Q_n, \Theta]$, $s_n(d_n(\theta) \mid \theta) = \frac{1}{2}(\theta - \hat{p}_n)^2 - (p_n - \hat{p}_n Q_n) \geq Q_n^2/2 + \hat{p}_n Q_n - p_n > 0$ by eq. (EC.92). In other words, all customers of types $\theta \quad [Q_n, \Theta]$ choose to subscribe. Thus, the continuity of s_n implies $\bar{\theta}_n < Q_n$: setting $s_n(d_n(\theta) \mid \theta)$ in eq. (EC.90) to be zero, we have $\bar{\theta}_n = \sqrt{2p_n}$.

Second, consider $Q_n^2/2 \leq p_n < \hat{p}_n Q_n + Q_n^2/2$. For any customer of type $\theta \quad [0, Q_n)$, $s_n(d_n(\theta) \mid \theta) = \frac{1}{2}\theta^2 - p_n \leq \frac{1}{2}\theta^2 - \frac{1}{2}Q_n^2 < 0$ by eq. (EC.90). In other words, no customers of types $\theta \quad [0, Q_n)$ choose to subscribe. For any customer of type $\theta \quad [\hat{p}_n + Q_n, \Theta]$, $s_n(d_n(\theta) \mid \theta) = \frac{1}{2}(\theta - \hat{p}_n)^2 - (p_n - \hat{p}_n Q_n) \geq \frac{1}{2}Q_n^2 - (p_n - \hat{p}_n Q_n) > 0$ by eq. (EC.92). In other words, all customers of types $\theta \quad [\hat{p}_n + Q_n, \Theta]$ choose to subscribe. Thus, the continuity of s_n implies $Q_n \leq \bar{\theta}_n < \hat{p}_n + Q_n$: setting $s_n(d_n(\theta) \mid \theta)$ in eq. (EC.91) to be zero, we have $\bar{\theta}_n = p_n/Q_n + Q_n^2/2$.

At last, consider $\hat{p}_n Q_n + Q_n^2/2 \leq p_n \leq \frac{1}{2}(\Theta - \hat{p}_n)^2 + \hat{p}_n Q_n$. For any customer of type $\theta \quad [0, Q_n)$, $s_n(d_n(\theta) \mid \theta) = \frac{1}{2}\theta^2 - p_n \leq \frac{1}{2}\theta^2 - \frac{1}{2}Q_n^2 - \hat{p}_n Q_n \leq \frac{1}{2}\theta^2 - \frac{1}{2}Q_n^2 < 0$ by eq. (EC.90). In other words, no customers of types $\theta \quad [0, Q_n)$ choose to subscribe. For any customer of type $\theta \quad [Q_n, \hat{p}_n + Q_n)$, $s_n(d_n(\theta) \mid \theta) = \theta Q_n - \frac{1}{2}Q_n^2 - p_n < (\hat{p}_n + Q_n)Q_n - \frac{1}{2}Q_n^2 - p_n = \hat{p}_n Q_n + Q_n^2/2 - p_n \leq 0$ by eq. (EC.91). In other words, no customers of types $\theta \quad [Q_n, \hat{p}_n + Q_n)$ choose to subscribe. Thus, the continuity of s_n implies $\hat{p}_n + Q_n \leq \bar{\theta}_n \leq \Theta$: setting $s_n(d_n(\theta) \mid \theta)$ in eq. (EC.92) to be zero, we have $\bar{\theta}_n = \hat{p}_n + \sqrt{2(p_n - \hat{p}_n Q_n)}$.

Proof of Corollary 2. By the proof of Proposition 4, we know the service provider earns the optimal profit when $\hat{p}_f Q_n + Q_n^2/2 \leq p_n \leq \frac{1}{2}(\Theta - \hat{p}_n)^2 + \hat{p}_n Q_n$. Therefore, $\bar{\theta}_n$ is given by Eq. (EC.92) in the proof of Lemma 4, which implies that $\bar{\theta}_n \geq \hat{p}_n + Q_n$. Moreover, Lemma 3 indicates $d_n(\theta) = \theta - \hat{p}_n \geq Q_n$ for any $\theta \quad [\bar{\theta}_n, \Theta]$.

Proof of Lemma 5. If allowing all to trade in the same resale market, by Lemma 1, $d_{s_k}(\theta) = \max\{\theta - \hat{p}_s, 0\}$. Thus, $s_{s_i}(d_{s_i}(\theta) \mid \theta) - s_{s_j}(d_{s_j}(\theta) \mid \theta) = (-p_{s_i} + \hat{p}_{s_i} Q_{s_i}) - (-p_{s_j} + \hat{p}_{s_j} Q_{s_j})$ by Eq. (TS.2). Therefore, all subscribers will choose the same tier k , where $k = \underset{k=1,2\ldots,K}{\arg\max} -p_{s_k} + \hat{p}_{s_k} Q_{s_k}$. In other words, offering a menu of sharing contracts equals to offering one contract. Therefore, restricting trading to subscribers of the same tier can generate a higher optimal revenue.

Proof of Proposition 7. $\overline{\Pi}_s \leq \overline{\Pi}_n$ is straightforward since Proposition 6 reveals that the two-part tariff yields the same revenue as the optimal sharing contract but is a special case of the nonlinear contract.

Proof of Proposition 8. Proposition TS1 shows that the service provider may choose (p_s, Q_s) such that either sharing with speculators or sharing without speculators takes place. We shall first show that sharing without speculators yields no less revenue than sharing with speculators.

Consider a given sharing contract (p_s, Q_s) so that $0 \leq Q_s < \overline{Q}_s$. In this case, sharing with speculators occurs by Proposition TS1(i). Consider the revenue function $\Pi_s(p_s, Q_s) = p_s \bar{F}(\bar{\theta}_s)$ and note that $\bar{F}(\bar{\theta}_s) = \left(\int_{\hat{p}_s - w_u Q_s}^{\hat{p}_s + Q_s} \frac{\theta - \hat{p}_s + w_u Q_s}{1 + w_u} f(\theta) \mathrm{d}\theta + \int_{\hat{p}_s + Q_s}^{\Theta} \frac{\theta - \hat{p}_s + w_o Q_s}{1 + w_o} f(\theta) \mathrm{d}\theta \right) \Big/ Q_s$ by (TS.18). Thus, we write

$$\Pi_s(p_s, Q_s) = p_s \bar{F}(\bar{\theta}_s) = p_s \left(\int_{\hat{p}_s - w_u Q_s}^{\hat{p}_s + Q_s} \frac{\theta - \hat{p}_s + w_u Q_s}{1 + w_u} f(\theta) \mathrm{d}\theta + \int_{\hat{p}_s + Q_s}^{\Theta} \frac{\theta - \hat{p}_s + w_o Q_s}{1 + w_o} f(\theta) \mathrm{d}\theta \right) \Big/ Q_s. \tag{EC.93}$$

Recall $\int_{\hat{p}_s - w_u Q_s}^{\hat{p}_s + Q_s} \frac{\theta - \hat{p}_s + w_u Q_s}{1 + w_u} f(\theta) \mathrm{d}\theta + \int_{\hat{p}_s + Q_s}^{\Theta} \frac{\theta - \hat{p}_s + w_o Q_s}{1 + w_o} f(\theta) \mathrm{d}\theta$ decreases in $\hat{p}_s$ and $\hat{p}_s \geq p_s/Q_s + w_u Q_s/2$ by Lemma TS7(i). Therefore, (EC.93) indicates

$$\Pi_s(p_s, Q_s) \leq (p_s/Q_s) \bar{F}(p_s/Q_s - w_u Q_s/2) \overline{Q}_s, \tag{EC.94}$$

where $\overline{Q}_s$ is defined in Proposition TS1.

We next show that there exists a sharing contract under which sharing without speculators occurs and the resulting revenue equals to $(p_s/Q_s) \bar{F}(p_s/Q_s - w_u Q_s/2) \overline{Q}_s$. Therefore, the maximum revenue by inducing sharing without speculators is no less than that with speculators. Denote $\hat{p}_{\underline{s}}$ as the market-clearing price and $\bar{\theta}_{\underline{s}}$ as the subscribing threshold, i.e., $s_{\underline{s}}\left(d_{\underline{s}}(\bar{\theta}_{\underline{s}}) | \bar{\theta}_{\underline{s}}\right) = \frac{(\bar{\theta}_{\underline{s}} - \hat{p}_{\underline{s}} + w_u Q_{\underline{s}})^2}{2(1 + w_u)} - \frac{1}{2} w_u Q_{\underline{s}}^2 - p_{\underline{s}} + \hat{p}_{\underline{s}} Q_{\underline{s}} = 0$ under $(p_{\underline{s}}, Q_{\underline{s}})$. It can be shown that the market clearing equation (TS.19) is achieved at $\bar{\theta}_s = \hat{p}_{\underline{s}} - w_u Q_s$ and $\hat{p}_s = \hat{p}_{\underline{s}}$ under $(p_{\underline{s}}, Q_{\underline{s}})$. Moreover, $\bar{\theta}_s = \hat{p}_{\underline{s}} - w_u Q_s$ and $\hat{p}_s = \hat{p}_{\underline{s}}$ must be the only solution due to the uniqueness of the equilibrium. Thus, $\bar{\theta}_{\underline{s}} = \hat{p}_{\underline{s}} - w_u Q_{\underline{s}}$ and $s_{\underline{s}}\left(d_{\underline{s}}(\bar{\theta}_{\underline{s}}) | \bar{\theta}_{\underline{s}}\right) = -\frac{1}{2} w_u Q_{\underline{s}}^2 - p_{\underline{s}} + \hat{p}_{\underline{s}} Q_{\underline{s}} = 0$, which imply $\hat{p}_{\underline{s}} = p_{\underline{s}}/Q_{\underline{s}} + w_u Q_{\underline{s}}/2$ and $\bar{\theta}_{\underline{s}} = \hat{p}_{\underline{s}} - w_u Q_{\underline{s}} = p_{\underline{s}}/Q_{\underline{s}} - w_u Q_{\underline{s}}/2$. Recall $p_{\underline{s}}/Q_{\underline{s}} = p_s/Q_s$ and $Q_{\underline{s}} = \overline{Q}_s$. Now consider the revenue under $(p_{\underline{s}}, Q_{\underline{s}})$

$$\begin{aligned} \Pi_s(p_{\underline{s}}, Q_{\underline{s}}) &= p_{\underline{s}} \bar{F}(\bar{\theta}_{\underline{s}}) = (p_{\underline{s}}/Q_{\underline{s}}) \bar{F}(p_{\underline{s}}/Q_{\underline{s}} - w_u Q_{\underline{s}}/2) Q_{\underline{s}} \\ &= (p_s/Q_s) \bar{F}(p_s/Q_s - w_u \overline{Q}_s/2) \overline{Q}_s > (p_s/Q_s) \bar{F}(p_s/Q_s - w_u Q_s/2) \overline{Q}_s, \end{aligned} \tag{EC.95}$$

where the inequality is due to $Q_s < \overline{Q}_s$. Putting (EC.94) and (EC.95) together, we claim that sharing without speculators must yield a higher revenue than with speculators.

At last, we solve for the optimal contract $(p_{\underline{s}}, Q_{\underline{s}})$. Since $p_{\underline{s}} \geq 0$ and $Q_{\underline{s}} \geq 0$, the optimal solution is either on the boundary or a stationary point. However, $\Pi_s(p_s, Q_s) = 0$ for $p_s = 0$ or $Q_s = 0$ and

$\Pi_s(p_s, Q_s) > 0$ for $p_s > 0$ and $Q_s > 0$. The optimal solution must arise at a stationary point. Consider the first-order conditions of (7)

$$\begin{cases} \dfrac{\partial \Pi_s}{\partial p_s} = \bar{F}(\bar{\theta}_s) - p_s f(\bar{\theta}_s)\dfrac{\partial \bar{\theta}_s}{\partial p_s} = 0 \\ \dfrac{\partial \Pi_s}{\partial Q_s} = -p_s f(\bar{\theta}_s)\dfrac{\partial \bar{\theta}_s}{\partial Q_s} = 0 \end{cases} \qquad \begin{cases} \dfrac{\partial \bar{\theta}_s}{\partial p_s} = \dfrac{\bar{F}(\bar{\theta}_s)}{p_s f(\bar{\theta}_s)} \\ \dfrac{\partial \bar{\theta}_s}{\partial Q_s} = 0 \end{cases}, \tag{EC.96}$$

which demonstrates the connection of the optimal contact with the subscribing threshold $\bar{\theta}_s$. We thus explore the properties of $\bar{\theta}_s$, which arises together with the market clearing price $\hat{p}_s$ via (TS.19). Recall $\frac{(\theta - \hat{p}_s + w_u Q_s)^2}{2(1+w_u)} - \frac{1}{2} w_u Q_s^2 - p_s + \hat{p}_s Q_s = 0$ implies $\hat{p}_s = \bar{\theta}_s + \sqrt{(1+w_u)Q_s^2 - 2(1+w_u)\bar{\theta}_s Q_s + 2(1+w_u)p_s} - Q_s$. Then, taking the first derivative of both sides of (TS.19) with respect to p_s and Q_s respectively, we have

$$-\frac{\bar{\theta}_s - \hat{p}_s - Q_s}{1+w_u} f(\bar{\theta}_s)\frac{\partial \bar{\theta}_s}{\partial p_s} - \left(\int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} \frac{1}{1+w_u} f(\theta)\mathrm{d}\theta + \int_{\hat{p}_s + Q_s}^{\Theta} \frac{1}{1+w_o} f(\theta)\mathrm{d}\theta \right)\frac{\partial \hat{p}_s}{\partial p_s} = 0, \tag{EC.97}$$

where $\frac{\partial \hat{p}_s}{\partial p_s} = \frac{\partial \bar{\theta}_s}{\partial p_s} - \sqrt{\frac{(1+w_u)\left(Q_s \frac{\partial \bar{\theta}_s}{\partial p_s} - 1\right)}{(1+w_u)Q_s^2 - 2(1+w_u)\bar{\theta}_s Q_s + 2(1+w_u)p_s}}$. And

$$-\frac{\bar{\theta}_s - \hat{p}_s - Q_s}{1+w_u} f(\bar{\theta}_s)\frac{\partial \bar{\theta}_s}{\partial Q_s} - \int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} \frac{\frac{\partial \hat{p}_s}{\partial Q_s} - w_u}{1+w_u} f(\theta)\mathrm{d}\theta - \int_{\hat{p}_s + Q_s}^{\Theta} \frac{\frac{\partial \hat{p}_s}{\partial Q_s} - w_o}{1+w_o} f(\theta)\mathrm{d}\theta = \bar{F}(\bar{\theta}_s), \tag{EC.98}$$

where $\frac{\partial \hat{p}_s}{\partial Q_s} = \frac{\partial \bar{\theta}_s}{\partial Q_s} - \sqrt{\frac{(1+w_u)\left(Q_s \frac{\partial \bar{\theta}_s}{\partial Q_s} + \bar{\theta}_s - Q_s\right)}{(1+w_u)Q_s^2 - 2(1+w_u)\bar{\theta}_s Q_s + 2(1+w_u)p_s}} - 1$. Note $\dfrac{\partial \bar{\theta}_s}{\partial Q_s} = 0$, we rewrite (EC.98) into $\sqrt{\frac{(1+w_u)\left(\bar{\theta}_s - Q_s\right)}{(1+w_u)Q_s^2 - 2(1+w_u)\bar{\theta}_s Q_s + 2(1+w_u)p_s}} \left(\int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} \frac{1}{1+w_u} f(\theta)\mathrm{d}\theta + \int_{\hat{p}_s + Q_s}^{\Theta} \frac{1}{1+w_o} f(\theta)\mathrm{d}\theta \right) = 0$. This implies $\bar{\theta}_s = Q_s$ and $\hat{p}_s = \sqrt{(1+w_u)(2p_s - {Q_s}^2)}$. Plugging $\bar{\theta}_s = Q_s$ and $\dfrac{\partial \bar{\theta}_s}{\partial p_s} = \dfrac{\bar{F}(\bar{\theta}_s)}{p_s f(\bar{\theta}_s)}$ into (TS.19) and (EC.97), we get

$$Q_s \bar{F}(Q_s) = \int_{Q_s}^{\hat{p}_s + Q_s} \frac{\theta - \hat{p}_s + w_u Q_s}{1+w_u} f(\theta)\mathrm{d}\theta + \int_{\hat{p}_s + Q_s}^{\Theta} \frac{\theta - \hat{p}_s + w_o Q_s}{1+w_o} f(\theta)\mathrm{d}\theta$$

and

$$\left(\int_{Q_s}^{\hat{p}_s + Q_s} \frac{1}{1+w_u} f(\theta)\mathrm{d}\theta + \int_{\hat{p}_s + Q_s}^{\Theta} \frac{1}{1+w_o} f(\theta)\mathrm{d}\theta \right)\left(\frac{\bar{F}(Q_s)}{f(Q_s)}(\hat{p}_s - (1+w_u)Q_s) + \frac{{\hat{p}_s}^2 + (1+w_u){Q_s}^2}{2} \right) = \frac{{\hat{p}_s}^2 \bar{F}(Q_s)}{1+w_u}.$$

Proof of Proposition 9. By Lemma TS9, if $p_n > \frac{(\Theta - \hat{p}_n + w_o Q_n)^2}{2(1+w_o)} + \hat{p}_n Q_n - \frac{w_o Q_n^2}{2}$, no customers subscribe and $\Pi(p_n, Q_n, \hat{p}_n) = 0$. We thus only need to consider $0 \le p_n \le \frac{(\Theta - \hat{p}_n + w_o Q_n)^2}{2(1+w_o)} + \hat{p}_n Q_n - \frac{w_o Q_n^2}{2}$. Specifically, we deliberate three cases: (a) $0 \le p_n < \frac{Q_n^2}{2}$; (b) $\frac{Q_n^2}{2} \le p < \hat{p}_n Q_n^2 + \frac{Q_n^2}{2}$; and (c) $\hat{p}_n Q_n^2 + \frac{Q_n^2}{2} \le p_n \le \frac{(\Theta - \hat{p}_n + w_o Q_n)^2}{2(1+w_o)} + \hat{p}_n Q_n - \frac{w_o Q_n^2}{2}$. We shall show that case (c) yields more profit than the other two cases and shall characterize the optimal solutions from case (c).

(a) $0 \le p_n < \frac{Q_n^2}{2}$. In this case, $\bar{\theta}_n = \sqrt{(1+w_u)(2p_n + w_u Q_n^2)} - w_u Q_n < Q_n$ by (TS.31). The revenue function in (12) and its derivative in Q_n can be written as

$$\Pi_n(p_n, Q_n, \hat{p}_n) = p_n \bar{F}(\bar{\theta}_n) + \hat{p}_n \int_{\hat{p}_n + Q_n}^{\Theta} (\theta - \hat{p}_n - Q_n) f(\theta)\mathrm{d}\theta \text{ and } \frac{\partial \Pi_n}{\partial Q_n} = -\hat{p}_n \bar{F}(\hat{p}_n + Q_n) \le 0.$$

It is obvious that $\Pi(p_n, Q_n, \hat{p}_n)$ is decreasing in Q_n for a given p_n if $0 \le p_n < \frac{Q_n^2}{2}$. Hence, $\Pi(p_n, Q_n, \hat{p}_n) < \Pi(p_n, Q_n = \sqrt{(1+w_u)(2p_n + w_u Q_n^2)} - w_u Q_n, \hat{p}_n)$ in this case. Therefore, the profit cannot be more than that when $\frac{1}{2}Q_n^2 \le p_n < \hat{p}_n Q_n + \frac{1}{2}Q_n^2$, i.e., case (b).

(b) $\frac{Q_n^2}{2} \leq p_n < \hat{p}_n Q_n + \frac{Q_n^2}{2}$. In this case, $\bar{\theta}_n = p_n/Q_n + Q_n/2$ by (TS.31) and $Q_n \leq \bar{\theta}_n < \hat{p}_n + Q_n$. We can rewrite the revenue function $\Pi_n(p_n, Q_n, \hat{p}_n)$ in (12) in terms of $(\bar{\theta}_n, Q_n, \hat{p}_n)$ as

$$\Pi_n(\bar{\theta}_n, Q_n, \hat{p}_n) = (\bar{\theta}_n Q_n - \frac{1}{2}Q_n^2)\bar{F}(\bar{\theta}_n) + \hat{p}_n \int_{\hat{p}_n + Q_n}^{\Theta} (\theta - \hat{p}_n - Q_n) f(\theta) \mathrm{d}\theta.$$

By the same analysis of Case (b) in proof of Proposition 4, we can show that the maximum value of $\Pi_n(\bar{\theta}_n, Q_n, \hat{p}_n)$ must be achieved at $\bar{\theta}_n = \hat{p}_n + Q_n$. In other words, the optimal solution must be a boundary point, which will be considered in case (c)

(c) $\hat{p}_n Q_n + \frac{Q_n^2}{2} \leq p_n \leq \frac{(\Theta - \hat{p}_n + w_o Q_n)^2}{2(1+w_o)} + \hat{p}_n Q_n - \frac{w_o Q_n^2}{2}$. By (TS.31), we have $\bar{\theta}_n = \hat{p}_n + \sqrt{(1+w_o)(2(p_n - \hat{p}_n Q_n) + w_o Q_n^2)} - w_o Q_n \geq \hat{p}_n + Q_n$, which is derived from setting (TS.34) to zero so that $p_n = \frac{(\bar{\theta}_n + w_o Q_n - \hat{p}_n)^2}{2(1+w_o)} - \frac{1}{2} w_o Q_n^2 + \hat{p}_n Q_n$. Rewrite the profit function (12),

$$\Pi_n(p_n, Q_n, \hat{p}_n) = \frac{(\bar{\theta}_n - \hat{p}_n)^2 + 2 w_o \bar{\theta}_n Q_n - w_o Q_n^2 - 2\hat{p}_n^2}{2(1+w_o)} \bar{F}(\bar{\theta}_n) + \hat{p}_n \int_{\bar{\theta}_n}^{\Theta} \frac{\theta}{1+w_o} f(\theta) \mathrm{d}\theta, \tag{EC.99}$$

and its derivative in Q_n,

$$\frac{\partial \Pi_n}{\partial Q_n} = \frac{w_o(\bar{\theta}_n - Q_n)}{1+w_o} \bar{F}(\bar{\theta}_n) \geq 0.$$

Hence, $\Pi(p_n, Q_n, \hat{p}_n)$ is increasing in Q_n and it is optimal to let $Q_n = \bar{\theta}_n - \hat{p}_n$. Plugging $Q_n = \bar{\theta}_n - \hat{p}_n$ into (EC.99), we get

$$\Pi_n(p_n, Q_n, \hat{p}_n) = (\frac{1}{2}\bar{\theta}_n^2 - \bar{\theta}_n \hat{p}_n - \frac{1}{2}\hat{p}_n^2 + \frac{w_o \bar{\theta}_n \hat{p}_n}{1+w_o}) \bar{F}(\bar{\theta}_n) + \hat{p}_n \int_{\bar{\theta}_n}^{\Theta} \frac{\theta}{1+w_o} f(\theta) \mathrm{d}\theta,$$

which only depends on $\bar{\theta}_n$ and $\hat{p}_n$. By the FOCs, the optimal nonlinear contract must satisfy

$$\begin{cases} \dfrac{\partial \Pi_n}{\partial \bar{\theta}_n} = \Big(\bar{\theta}_n - \dfrac{\hat{p}_n}{1+w_o}\Big) \bar{F}(\bar{\theta}_n) - \dfrac{1}{2}(\bar{\theta}_n^2 - \hat{p}_n^2) f(\bar{\theta}_n) = 0, & \text{(EC.100)} \\ \dfrac{\partial \Pi_n}{\partial \hat{p}_n} = \displaystyle\int_{\bar{\theta}_n}^{\Theta} \frac{\theta}{1+w_o} f(\theta) \mathrm{d}\theta - \Big(\frac{\bar{\theta}_n}{1+w_o} + \hat{p}_n\Big) \bar{F}(\bar{\theta}_n) = 0. & \text{(EC.101)} \end{cases}$$

Solving (EC.100) and (EC.101), we obtain

$$\frac{(1+w_o)^2 \bar{\theta}_n^2 - \Big(\frac{\int_{\bar{\theta}_n}^{\Theta} \theta f(\theta) \mathrm{d}\theta}{\bar{F}(\bar{\theta}_n)} - \bar{\theta}_n\Big)^2}{2\Big[(1+w_o)^2 \bar{\theta}_n - \frac{\int_{\bar{\theta}_n}^{\Theta} \theta f(\theta) \mathrm{d}\theta}{\bar{F}(\bar{\theta}_n)} + \bar{\theta}_n\Big]} = \frac{\bar{F}(\bar{\theta}_n)}{f(\bar{\theta}_n)}, \ \hat{p}_n = \frac{\int_{\bar{\theta}_n}^{\Theta} \theta f(\theta) \mathrm{d}\theta}{(1+w_o)\bar{F}(\bar{\theta}_n)} - \frac{\bar{\theta}_n}{1+w_o}.$$

Moreover, $p_n = \frac{(\bar{\theta}_n + w_o Q_n - \hat{p}_n)^2}{2(1+w_o)} - \frac{1}{2} w_o {Q_n}^2 + \hat{p}_n Q_n = \frac{(\bar{\theta}_n - \hat{p}_n)^2}{2} + \hat{p}_n(\bar{\theta}_n - \hat{p}_n)$ due to $Q_n = \bar{\theta}_n - \hat{p}_n$.

EC4. Proofs in Section EC1

Proof of Lemma EC1. It is obvious that $s(d \,|\, \theta)$ in (EC.1) is concave in d. Hence, if a customer has already subscribed to the service, i.e., $s(d \,|\, \theta) \geq 0$, we can derive her demand by the first order condition (FOC). Taking the derivative, we have

$$\frac{\partial s}{\partial d} = \theta - d = 0 \qquad d\ \ (\theta) = \theta.$$

Note that by definition of a bucket contract, $d \leq Q$. Thus, $d\ \ (\theta) = \min\{\theta, Q\}$. According to (EC.1), we can write $s(d\ \ (\theta) \,|\, \theta)$ as

$$s(d\ \ (\theta) \,|\, \theta) = \begin{cases} \dfrac{1}{2}\theta^2 - p, & \text{if } 0 \leq \theta < Q & \text{(EC.102)} \\ \theta Q - \dfrac{1}{2}Q^2 - p, & \text{if } Q \leq \theta \leq \Theta. & \text{(EC.103)} \end{cases}$$

If $p > Q^2/2 + \Theta Q$, $s(d^*(\theta = \Theta) \,|\, \theta = \Theta) < 0$. Since $s(d^*(\theta) \,|\, \theta)$ strictly increases in θ, no customers earn positive surplus and hence there will be no subscribers. On the other hand, if $0 \leq p \leq Q^2/2 + \Theta Q$, $s(d^*(\theta = 0) \,|\, \theta = 0) \leq 0$ and $s(d^*(\theta = \Theta) \,|\, \theta = \Theta) \geq 0$. Therefore, there exists a unique $\bar{\theta}$ such that $s(d^*(\theta) \,|\, \theta) = 0$ and customers subscribe to the service if and only if $\theta \geq \bar{\theta}$.

Next, we characterize $\bar{\theta}$. First, let us consider the case $0 \leq p < Q^2/2$. For any customer of type $\theta \in [Q, \Theta]$, $s(d^*(\theta) \,|\, \theta) = \theta Q - \frac{1}{2}Q^2 - p > \theta Q - Q^2 \geq 0$ by Eq. (EC.103). In other words, all customers of types $\theta \in [Q, \Theta]$ choose to subscribe. This implies $\bar{\theta} < Q$: setting $s(d^*(\theta) \,|\, \theta)$ in Eq. (EC.102) to be zero, we have $\bar{\theta} = \sqrt{2p}$. Second, consider $Q^2/2 \leq p \leq Q^2/2 + \Theta Q$. For any customer of type $\theta \in [0, Q)$, $s(d^*(\theta) \,|\, \theta) = \frac{1}{2}\theta^2 - p \leq \frac{1}{2}\theta^2 - \frac{1}{2}Q^2 < 0$ by Eq. (EC.102). In other words, no customers of types $\theta \in [0, Q)$ choose to subscribe. This implies $\bar{\theta} \geq Q$: setting $s(d^*(\theta) \,|\, \theta)$ in Eq. (EC.103) to be zero, we have $\bar{\theta} = p/Q + Q/2$ and $d^*(\theta) = \min\{\theta, Q\} = Q$ for $\theta \geq \bar{\theta}$.

Proof of Proposition EC1. By Lemma EC1, if $p > Q^2/2 + \Theta Q$, no customers subscribe and $\Pi(p, Q) = 0$. We thus only consider two cases: (i) $0 \leq p < Q^2/2$ and (ii) $Q^2/2 \leq p \leq Q^2/2 + \Theta Q$.

(i) $\mathbf{0 \leq p < Q^2/2}$. In this case, $\bar{\theta}(p, Q) = \sqrt{2p}$ by Lemma EC1 and $\Pi(p, Q) = p \cdot \bar{F}(\bar{\theta}(p, Q)) = p\bar{F}(\sqrt{2p})$ by (EC.2). Note that $\Pi(p, Q)$ is in fact a univariate function in p and we thus write it in short as $\Pi(p)$. Taking the first-order derivative, we have

$$\frac{\partial \Pi}{\partial p} = \bar{F}(\sqrt{2p}) - \frac{1}{2}\sqrt{2p}f(\sqrt{2p}) = 0 \quad \Leftrightarrow \quad \frac{\bar{F}(\sqrt{2p})}{\sqrt{2p}f(\sqrt{2p})} = \frac{1}{2}. \tag{EC.104}$$

We shall show that Eq. (EC.104) has a solution, denoted as p^*, and moreover $p^* = \arg\max_p \Pi(p)$. Since $F(\cdot)$ has an IFR, $\frac{\bar{F}(\sqrt{2p})}{\sqrt{2p}f(\sqrt{2p})}$ is decreasing in $\sqrt{2p}$. Note that $\lim_{p\to 0} \frac{\bar{F}(\sqrt{2p})}{\sqrt{2p}f(\sqrt{2p})} = \infty$ and $\lim_{p\to\infty} \frac{\bar{F}(\sqrt{2p})}{\sqrt{2p}f(\sqrt{2p})} = 0$, thus there exists a p^* such that $\frac{\bar{F}(\sqrt{2p^*})}{\sqrt{2p^*}f(\sqrt{2p^*})} = \frac{1}{2}$. For the optimality of p^*, let us consider $\partial\Pi/\partial p$. Since $F(\cdot)$ has an IFR, it is easy to see, from Eq. (EC.104), that $\partial\Pi/\partial p \geq 0$ if $0 \leq p \leq p^*$ and $\partial\Pi/\partial p < 0$ if $p > p^*$. In other words, $\Pi(p)$ is increasing for $p \leq p^*$ and decreasing for $p > p^*$. Thus, $p^* = \arg\max_p \Pi(p)$. Moreover, there are infinitely many optimal free allowance Q^*'s as long as $Q^* > \sqrt{2p^*}$ and customers subscribe if and only if $\theta \geq \bar{\theta}(p^*, Q^*) = \sqrt{2p^*}$.

(ii) $\mathbf{Q^2/2 \leq p \leq Q^2/2 + \Theta Q}$. In this case, $\bar{\theta}(p, Q) = p/Q + Q/2$ by Lemma EC1 and $\Pi(p, Q) = p \cdot \bar{F}(\bar{\theta}(p, Q)) = p\bar{F}(p/Q + Q/2)$ by (EC.2). Taking the first-order derivative w.r.t. Q gives

$$\frac{\partial \Pi(p, Q)}{\partial Q} = f(p/Q + Q/2)\left(\frac{p}{Q^2} - \frac{1}{2}\right) \geq 0,$$

which implies that $\Pi(p, Q)$ is increasing in Q when $Q^2/2 \leq p \leq Q^2/2 + \Theta Q$. Hence, we conclude that the optimal solution must be obtained when $p = Q^2/2$. We then can transform (EC.2) as follows

$$\max_{p,Q} \Pi(p, Q) = p \cdot \bar{F}\left(\bar{\theta}(p, Q)\right) = p\bar{F}(p/Q + Q/2) \quad \Rightarrow \quad \max_p \Pi\left(p, Q = \sqrt{2p}\right) = p\bar{F}\left(\sqrt{2p}\right). \tag{EC.105}$$

Following similar analysis as in case (i), we know that $\Pi(p, Q)$ is maximized when $p = p^*$ and $Q^* = \sqrt{2p^*}$, where p^* is the solution to (EC.104). Moreover, customers subscribe if and only if $\theta \geq \bar{\theta}(p^*, Q^*) = p^*/Q^* + Q^*/2 = Q^* = \sqrt{2p^*}$.

Combining the results in cases (i) and (ii), we conclude that the optimal price p^* is determined by (EC.3), $Q^* \geq \sqrt{2p^*}$, and $\bar{\theta}^* = \bar{\theta}(p^*, Q^*) = \sqrt{2p^*}$.

Proof of Proposition EC2. (i) We first show $\bar\theta_s \le \bar\theta$. By Lemma 2, the equilibrium under a sharing mechanism may have two forms: sharing with and without speculators. We shall show that $\bar\theta_s \le \bar\theta$ hold for both scenarios.

Sharing with speculators. If $\bar\theta_s = 0$, the result is trivial. If $\bar\theta_s > 0$, the proof of Lemma 2(i) reveals that $\bar\theta_s < \hat p_s = p_s/Q_s$. The result $\bar\theta_s \le \bar\theta$ holds if $\bar\theta \ge p/Q = p_s/Q_s$, which we shall show next. By Lemma EC1, if $p < Q^2/2$, $\bar\theta = \sqrt{2p}$, we have $\sqrt{2p}/(p/Q) = \sqrt{2}Q/\sqrt{p} > \sqrt{2}\sqrt{2p}/\sqrt{p} = 1$; If $Q^2/2 \le p \le Q^2/2 + \Theta Q$, $\bar\theta = p/Q + Q/2 \ge p/Q$.

Sharing without speculators. In this case, (EC.81) implies that $\frac{1}{2}(\bar\theta_s - \hat p_s)^2 = p_s - \hat p_s Q_s$. Moreover, Lemma 2(ii) shows $\bar\theta_s \ge \hat p_s$. Thus, we have $\bar\theta_s = \sqrt{2(p_s - \hat p_s Q_s)} + \hat p_s$. If $p < Q^2/2$, $\bar\theta = \sqrt{2p}$ by Lemma EC1 and $\bar\theta - \bar\theta_s = \sqrt{2p} - \sqrt{2(p_s - \hat p_s Q_s)} - \hat p_s = \sqrt{2p} - \sqrt{2(p - \hat p_s Q)} - \hat p_s = \frac{2\hat p_s Q}{\sqrt{2p} + \sqrt{2(p-\hat p_s Q)}} - \hat p_s$, where the second equality is due to $(p,Q) = (p_s, Q_s)$. Since $p < Q^2/2$ and $\hat p_s Q \ge \sqrt{0}$,

$$\bar\theta - \bar\theta_s = \frac{2\hat p_s Q}{\sqrt{2p} + \sqrt{2(p-\hat p_s Q)}} - \hat p_s > \frac{2\hat p_s \sqrt{2p}}{\sqrt{2p} + \sqrt{2(p-\hat p_s Q)}} - \hat p_s \ge \frac{2\hat p_s \sqrt{2p}}{\sqrt{2p} + \sqrt{2p}} - \hat p_s = 0 \Leftrightarrow \bar\theta_s < \bar\theta.$$

Consider $Q^2/2 \le p \le Q^2/2 + \Theta Q$ and apply the inequality of arithmetic and geometric means,

$$\bar\theta = p/Q + Q/2 = (p/Q - \hat p_s) + Q/2 + \hat p_s \ge \sqrt{2(p - \hat p_s Q)} + \hat p_s = \sqrt{2(p_s - \hat p_s Q_s)} + \hat p_s = \bar\theta_s,$$

where the third equality is due to the fact $(p,Q) = (p_s, Q_s)$.

Next, we show $\bar\theta_n \le \bar\theta$. We will consider two scenarios according to the values of p and Q. (a) If $p < Q^2/2$, $\bar\theta = \sqrt{2p}$ by Lemma EC1. Since $p_n = p$ and $Q_n = Q$, $p_n < Q_n^2/2$ as well. By Lemma 4, $p_n < Q_n^2/2$, $\bar\theta_n = \sqrt{2p_n} = \theta$. (b) If $Q^2/2 \le p \le Q^2/2 + \Theta Q$, $\bar\theta = p/Q + Q/2$ by Lemma EC1. However, $\bar\theta_n$ may have two forms according to Lemma 4. In case that $Q_n^2/2 \le p_n < \hat p_n Q_n + Q_n^2/2$, $\bar\theta_n = p_n/Q_n + Q_n/2$. Then, $\bar\theta_n = \bar\theta$. In case that $\hat p_n Q_n + Q_n^2/2 \le p_n \le \frac{1}{2}(\Theta - \hat p_n)^2 + \hat p_n Q_n$, $\bar\theta_n = \hat p_n + \sqrt{2(p_n - \hat p_n Q_n)}$. Taking the first-order derivative of $\bar\theta_n$ w.r.t. $\hat p_n$,

$$\frac{\partial \bar\theta_n}{\partial \hat p_n} = 1 - \frac{Q_n}{\sqrt{2(p_n - \hat p_n Q_n)}} \ge 1 - \frac{Q_n}{\sqrt{2(\hat p_n Q_n + Q_n^2/2 - \hat p_n Q_n)}} = 0,$$

where the inequality is due to $\hat p_n Q_n + Q_n^2/2 \le p_n$. Thus, $\bar\theta_n$ increases in $\hat p_n$, which implies that

$$\begin{aligned}
\bar\theta_n &= \hat p_n + \sqrt{2(p_n - \hat p_n Q_n)} \\
&\le p_n/Q_n - Q_n/2 + \sqrt{2(p_n - (p_n/Q_n - Q_n/2)Q_n)} \\
&= p_n/Q_n + Q_n/2 \\
&= p/Q + Q/2 \\
&= \bar\theta.
\end{aligned}$$

The inequality is because $\hat p_n \le p_n/Q_n - Q_n/2$ by $\hat p_n Q_n + Q_n^2/2 \le p_n$ and the second-last equality is due to the fact that $(p,Q) = (p_n, Q_n)$.

(ii) We first show $s_s(d_s(\theta) \mid \theta) \ge s(d\ (\theta) \mid \theta)$. Note we have proved that $\bar\theta_s \le \bar\theta$ in Proposition EC2(i). Thus, customers of type $\theta < \bar\theta$ subscribe to neither contracts, thus their surplus equals zero under both contracts. For customers of type $\bar\theta_s \le \theta < \bar\theta$, they only subscribe to the sharing contract but not to the bucket contract by Lemmas 1 and EC1, respectively. Therefore, $s_s(d_s(\theta) \mid \theta) \ge s(d\ (\theta) \mid \theta) = 0$.

Next, we consider customers of type $\theta \ge \bar\theta$ for two cases:

(a) $\bar{\theta} \leq \theta < \hat{p}_s$. In this case, $d\ (\theta) = \min\{\theta, Q\}$ and $d_s(\theta) = \max\{\theta - \hat{p}_s, 0\} = 0$. By (EC.1) and (4),

$$s(d\ (\theta) \mid \theta) = \theta d\ (\theta) - \frac{1}{2}(d\ (\theta))^2 - p \leq \theta d\ (\theta) - p < \hat{p}_s Q - p = s_s(d_s(\theta) \mid \theta),$$

where the last inequality is due to $\theta < \hat{p}_s$ and $d\ (\theta) \leq Q$.

(b) $\theta \geq \hat{p}_s$. In this case, $d\ (\theta) = \min\{\theta, Q\}$ and $d_s(\theta) = \max\{\theta - \hat{p}_s, 0\} = \theta - \hat{p}_s$. By (EC.1) and (4),

$$s(d\ (\theta) \mid \theta) = \theta d\ (\theta) - \frac{1}{2}(d\ (\theta))^2 - p = \theta d\ (\theta) - \hat{p}_s Q - \frac{1}{2}(d\ (\theta))^2 + \hat{p}_s Q - p.$$

Since $d\ (\theta) \leq Q$, then

$$s(d\ (\theta) \mid \theta) \leq \theta d\ (\theta) - \hat{p}_s d\ (\theta) - \frac{1}{2}(d\ (\theta))^2 + \hat{p}_s Q - p \leq \frac{1}{2}(\theta - \hat{p}_s)^2 + \hat{p}_s Q - p = s_s(d_s(\theta) \mid \theta),$$

where the last inequality is because $(\theta - \hat{p}_s) \cdot d\ (\theta) \leq \frac{1}{2}(\theta - \hat{p}_s)^2 + \frac{1}{2}(d\ (\theta))^2$.

We next show $s_n(d_n(\theta) \mid \theta) \geq s(d\ (\theta) \mid \theta)$. Note we have proved that $\bar{\theta}_n \leq \bar{\theta}$ in Proposition EC2(i). Thus, customers of type $\theta < \bar{\theta}$ subscribe to neither contracts, i.e., $s_n(d_n(\theta) \mid \theta) = s(d\ (\theta) \mid \theta) = 0$. For customers of type $\bar{\theta}_n \leq \theta < \bar{\theta}$, they only subscribe to the nonlinear contract by Lemma 4 but not to the bucket contract by Lemma EC1. Therefore, $s_n(d_n(\theta) \mid \theta) \geq s(d\ (\theta) \mid \theta) = 0$. For customers of type $\theta \geq \bar{\theta}$, we consider three cases:

(a) $\bar{\theta} \leq \theta < Q_n$. In this case, $d\ (\theta) = \min\{\theta, Q\} = \theta$ by Proposition EC1(ii) and $d_n(\theta) = \theta$ by (10). Since $(p, Q) = (p_n, Q_n)$ and $\hat{p}_n \geq 0$, we have, from (EC.1) and (9), that

$$s(d\ (\theta) \mid \theta) = \theta d\ (\theta) - \frac{1}{2}(d\ (\theta))^2 - p = \theta d_n(\theta) - \frac{1}{2}(d_n(\theta))^2 - p_n - \hat{p}_n(d_n(\theta) - Q)^+ = s_n(d_n(\theta) \mid \theta).$$

(b) $Q_n \leq \theta < \hat{p}_n + Q_n$. In this case, $d\ (\theta) = \min\{\theta, Q\} = Q$ by Proposition EC1(ii) and $d_n(\theta) = Q_n$ by (10). Since $(p, Q) = (p_n, Q_n)$ and $\hat{p}_n \geq 0$, we have, from (EC.1) and (9), that

$$s(d\ (\theta) \mid \theta) = \theta d\ (\theta) - \frac{1}{2}(d\ (\theta))^2 - p = \theta d_n(\theta) - \frac{1}{2}(d_n(\theta))^2 - p_n - \hat{p}_n(d_n(\theta) - Q)^+ = s_n(d_n(\theta) \mid \theta).$$

(c) $\hat{p}_n + Q_n \leq \theta \leq \Theta$. In this case, $d\ (\theta) = \min\{\theta, Q\} = Q$ by Proposition EC1(ii) and $d_n(\theta) = \theta - \hat{p}_n$ by (10). We have, from (EC.1) and (9), that

$$s(d\ (\theta) \mid \theta) = \theta d\ (\theta) - \frac{1}{2}(d\ (\theta))^2 - p = \theta Q - \frac{1}{2}Q^2 - p,$$

and

$$s_n(d_n(\theta) \mid \theta) = \theta d_n(\theta) - \frac{1}{2}(d_n(\theta))^2 - p_n - \hat{p}_n(d_n(\theta) - Q)^+ = \frac{1}{2}\theta^2 - \theta\hat{p}_n + \frac{1}{2}\hat{p}_n^2 + \hat{p}_n Q_n - p_n.$$

Taking the difference,

$$\begin{aligned} s_n(d_n(\theta) \mid \theta) - s(d\ (\theta) \mid \theta) &= \frac{1}{2}\theta^2 - (\hat{p}_n + Q)\theta + \frac{1}{2}\hat{p}_n^2 + \hat{p}_n Q_n + \frac{1}{2}Q^2 + p - p_n \\ &= \frac{1}{2}\theta^2 - (\hat{p}_n + Q)\theta + \frac{1}{2}(\hat{p}_n + Q)^2 \\ &= \frac{1}{2}(\theta - \hat{p}_n - Q)^2 \\ &\geq 0, \end{aligned}$$

where the second equality is because $(p, Q) = (p_n, Q_n)$.

(iii) By (EC.2) and (7), $\Pi(p, Q) = p\bar{F}(\bar{\theta})$ and $\Pi_s(p_s, Q_s) = p_s\bar{F}(\bar{\theta}_s)$. Recall from Proposition EC2(i) that $\bar{\theta}_s \leq \bar{\theta}$. Hence, $\Pi_s(p_s, Q_s) \geq \Pi(p, Q)$ when $(p, Q) = (p_s, Q_s)$. By (12), $\Pi_n(p_n, Q_n, \hat{p}_n) = p_n\bar{F}(\bar{\theta}_n) +$

$\hat{p}_n \cdot \int_{\max\{\bar{\theta}_n, \hat{p}_n + Q_n\}}^{\Theta} [(\theta - \hat{p}_n) - Q_n] \mathrm{d}F(\theta)$. Since $\bar{\theta}_n \leq \bar{\theta}$ from (i) and $\hat{p}_n \geq 0$, $\Pi_n(p_n, Q_n, \hat{p}_n) \geq \Pi(p, Q)$ when $(p, Q) = (p_n, Q_n)$.

Proof of Proposition EC3. (i) We first show $\bar{\theta}_s \leq \bar{\theta}$. By Proposition 3, we have $\bar{\theta}_s = Q_s$. Thus, $\bar{\theta}_s$ also satisfies (8), i.e., $\dfrac{\bar{F}(\bar{\theta}_s)}{f(\bar{\theta}_s)} = \frac{\int_{\bar{\theta}_s}^{\Theta} \theta f(\theta) \mathrm{d}\theta}{2\bar{F}(\bar{\theta}_s)}$, which implies that

$$\frac{\bar{F}(\bar{\theta}_s)}{\bar{\theta}_s f(\bar{\theta}_s)} = \frac{\int_{\bar{\theta}_s}^{\Theta} \theta f(\theta) \mathrm{d}\theta}{2\bar{\theta}_s \bar{F}(\bar{\theta}_s)} \geq \frac{1}{2} = \frac{\bar{F}(\bar{\theta}\ )}{\bar{\theta}\ f(\bar{\theta}\ )}. \tag{EC.106}$$

The inequality in (EC.106) is due to $\int_{\bar{\theta}_s}^{\Theta} \theta f(\theta) \mathrm{d}\theta \geq \bar{\theta}_s \bar{F}(\bar{\theta}_s)$ and the last equality results from $\dfrac{\bar{F}(\bar{\theta}\ )}{\bar{\theta}\ f(\bar{\theta}\ )} = \dfrac{1}{2}$ in Proposition EC1.

Recall that $F(\cdot)$ has an increasing generalized failure rate, i.e., $\frac{\bar{F}(x)}{x f(x)}$ is decreasing in x. Consequently, (EC.106) implies that $\bar{\theta}_s \leq \bar{\theta}$. By Proposition 4(i), $\dfrac{\bar{F}(\bar{\theta}_n)}{f(\bar{\theta}_n)} = \frac{\int_{\bar{\theta}_n}^{\Theta} \theta f(\theta) \mathrm{d}\theta}{2\bar{F}(\bar{\theta}_n)}$. Applying analogous analysis as above, it is easy to see $\bar{\theta}_n \leq \bar{\theta}$.

(ii) This result is straightforward from Proposition EC2(iii).

(iii) Recall that $\bar{\theta}\ = Q\ $ and $\bar{\theta}_s = Q_s$ from Propositions EC1 and 3, respectively. Since $\bar{\theta}_s \leq \bar{\theta}$ from Proposition EC3(i), we have $Q_s \leq Q$. Moreover, we have $Q_n \leq \bar{\theta}_n - \hat{p}_n \leq \bar{\theta}_n$ from (13). Since $\bar{\theta}_n \leq \bar{\theta}$ from Proposition EC3(i), we have $Q_n \leq Q$.

We now write out the total usage under each optimal contract. The total usage under the optimal bucket contract can be written as $\int_{\bar{\theta}}^{\Theta} d\ (\theta) f(\theta) d\theta$. Since $d\ (\theta) = \min\{\theta, Q\}$ by Proposition EC1, we can find an upper bound of the total usage, i.e.,

$$\int_{\bar{\theta}}^{\Theta} d\ (\theta) f(\theta) d\theta \leq Q\ \bar{F}(\bar{\theta}\ ) = Q\ \bar{F}(Q\ ).\text{[14]} \tag{EC.107}$$

Under the sharing contract, all goods sold by the provider will be consumed due to the market clearing mechanism. The total usage under the optimal sharing contract is

$$\int_{\bar{\theta}_s}^{\Theta} d_s(\theta) f(\theta) d\theta = Q_s \bar{F}(\bar{\theta}_s) = Q_s \bar{F}(Q_s), \tag{EC.108}$$

where the last equality is because $\bar{\theta}_s = Q_s$ by Proposition 3. The total usage under the optimal nonlinear contract can be written as

$$\int_{\bar{\theta}_n}^{\Theta} d_n(\theta) f(\theta) d\theta = \int_{\bar{\theta}_n}^{\Theta} (\theta - \hat{p}_n) f(\theta) d\theta = \int_{\bar{\theta}_n}^{\Theta} \theta dF(\theta) - \int_{\bar{\theta}_n}^{\Theta} \theta f(\theta) d\theta + \bar{\theta}_n \bar{F}(\bar{\theta}_n) = \bar{\theta}_n \bar{F}(\bar{\theta}_n), \tag{EC.109}$$

where the first equality stems from Corollary 2 and the second equality is due to $\hat{p}_n = \int_{\bar{\theta}_n}^{\Theta} \theta f(\theta) \mathrm{d}\theta / \bar{F}(\theta_n) - \bar{\theta}_n$ in Proposition 4.

In order to show $\int_{\bar{\theta}_s}^{\Theta} d_s(\theta) f(\theta) d\theta \geq \int_{\bar{\theta}}^{\Theta} d\ (\theta) f(\theta) d\theta$ and $\int_{\bar{\theta}_n}^{\Theta} d_n(\theta) f(\theta) d\theta \geq \int_{\bar{\theta}}^{\Theta} d\ (\theta) f(\theta) d\theta$. We need to establish the following lemma.

[14] Note that there exist multiple optimal bucket contracts and we choose the smallest optimal allowance for the proof. In this case, $\bar{\theta}\ = Q\ $ by Proposition EC1.

Lemma EC2. *If $F(x)$ has an increasing failure rate (IFR), then $x\bar{F}(x)$ has an inverted U-shape in $[0,\Theta]$, i.e., there exists a unique x such that $x\bar{F}(x)$ increases in $[0,x\ ]$ and decreases in $[x\ ,\Theta]$.*

Proof of Lemma EC2. Consider the derivative $\left(x\bar{F}(x)\right) = \bar{F}(x) - xf(x)$. Since $\left(x\bar{F}(x)\right)_{x=0} = 1$ and $\left(x\bar{F}(x)\right)_{x=\Theta} < 0$. Therefore, there exists an $x\ > 0$ such that $\left(x\bar{F}(x)\right)_{x=x} = \bar{F}(x\ ) - x\ f(x\ ) = 0$, or equivalently, $\frac{\bar{F}(x\ )}{x\ f(x\ )} = 1$. Moreover, since $F(\cdot)$ has an IFR, i.e., $\frac{\bar{F}(x)}{xf(x)}$ is decreasing in x, such an x must also be unique. As a result, for any $0 < x < x\ $ $(x > x\ )$, $\frac{\bar{F}(x)}{xf(x)} \geq (\leq) \frac{\bar{F}(x\ )}{x\ f(x\ )} = 1$, which implies that $\left(x\bar{F}(x)\right) = \bar{F}(x) - xf(x) \geq (\leq) 0$.

To show $\int_{\bar{\theta}_s}^{\Theta} d_s(\theta)f(\theta)d\theta \geq \int_{\bar{\theta}}^{\Theta} d\ (\theta)f(\theta)d\theta$, (EC.107) and (EC.108) suggest that it is sufficient to demonstrate $Q_s\bar{F}(Q_s) \geq Q\ \bar{F}(Q\ )$, which will hold if $x\ \leq Q_s \leq Q$ due to Lemma EC2. Since we have proved $Q_s \leq Q$, we only need to show $x\ \leq Q_s$. Proposition 3 indicates $\frac{\bar{F}(Q_s)}{f(Q_s)} = \int_{Q_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta/(2\bar{F}(Q_s))$ and we also have $\int_{\theta_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta/\bar{F}(\theta_s) = (Q_s + \hat{p}_s)$ from (EC.17). As a result,

$$\frac{\bar{F}(Q_s)}{f(Q_s)} = \int_{Q_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta \Big/ (2\bar{F}(Q_s)) = (Q_s + \hat{p}_s)/2 \leq Q_s, \tag{EC.110}$$

where the inequality is due to the fac that $\theta_s = Q_s$ by Proposition 3 and $\theta_s \geq \hat{p}_s$ by Lemma 2. Since $\frac{\bar{F}(x\ )}{x\ f(x\ )} = 1$ and $\frac{\bar{F}(x)}{xf(x)}$ decreases in x, we thus conclude $x\ \leq Q_s$ due to $\frac{\bar{F}(Q_s)}{Q_s f(Q_s)} \leq 1$ from (EC.110).

To show $\int_{\bar{\theta}_n}^{\Theta} d_n(\theta)f(\theta)d\theta \geq \int_{\bar{\theta}}^{\Theta} d\ (\theta)f(\theta)d\theta$, (EC.107) and (EC.109) suggest that it is sufficient to demonstrate $\bar{\theta}_n\bar{F}(\bar{\theta}_n) \geq Q\ \bar{F}(Q\ )$, which will hold if $x\ \leq \bar{\theta}_n \leq Q$ due to Lemma EC2. Recall that $\bar{\theta}_n \leq \bar{\theta}$ by Proposition EC3(i) and $\bar{\theta}\ = Q$ by Proposition EC1, thus it is obvious that $\bar{\theta}_n \leq Q$. Then, we only need to show $x\ \leq \bar{\theta}_n$. By Proposition 4,

$$\frac{\bar{F}(\bar{\theta}_n)}{f(\bar{\theta}_n)} = \frac{\int_{Q_n}^{\Theta} \theta f(\theta)\mathrm{d}\theta}{2\bar{F}(\bar{\theta}_n)} = (\bar{\theta}_n + \hat{p}_n)/2 \leq \bar{\theta}_n,$$

where the second equality is due to (EC.35), and the inequality is due to $\bar{\theta}_n - \hat{p}_n \geq 0$ from Proposition 4. With similar analysis applied to (EC.110), it follows that $x\ \leq \bar{\theta}_n$.

Proof of Proposition EC4. Note $\overline{\Pi}_s \leq \overline{\Pi}_n$ from Proposition 7, then we should prove $\overline{\Pi}\ \leq \overline{\Pi}_s$. We first prove a property of the optimal bucket contract.

Lemma EC3. *There must exist an optimal bucket contract such that $Q\ \geq \theta + \epsilon_\theta^u$ for any subscriber of a given type θ.*

Proof of Lemma EC3. Let $(p\ ,Q\ )$ be an optimal bucket contract. Pick an arbitrary subscriber of the bucket contract $(p\ ,Q\ )$ and assume she is type θ. For a realized ϵ_θ, we can write her optimal demand and utility as

$$d\ (\theta+\epsilon_\theta) = \begin{cases} 0, & \text{if } \theta+\epsilon_\theta < 0 \\ \theta+\epsilon_\theta, & \text{if } 0 \leq \theta+\epsilon_\theta < Q \\ Q\ , & \text{if } \theta+\epsilon_\theta \geq Q \end{cases} \quad \text{and } u\ (d\ \mid \theta+\epsilon_\theta) = \begin{cases} 0, & \text{if } \theta+\epsilon_\theta < 0 \\ (\theta+\epsilon_\theta)^2/2, & \text{if } 0 \leq \theta+\epsilon_\theta < Q \\ (\theta+\epsilon_\theta)Q\ - Q^{\ 2}/2, & \text{if } \theta+\epsilon_\theta \geq Q\ , \end{cases}$$

respectively. Since $c(d\ \mid \theta+\epsilon_\theta) = p$, we have

$$s\ (d\ \mid \theta+\epsilon_\theta) = u\ (d\ \mid \theta+\epsilon_\theta) - c(d\ \mid \theta+\epsilon_\theta) = \begin{cases} -p\ , & \text{if } \theta+\epsilon_\theta < 0 & \text{(EC.111a)} \\ (\theta+\epsilon_\theta)^2/2 - p\ , & \text{if } 0 \leq \theta+\epsilon_\theta < Q & \text{(EC.111b)} \\ (\theta+\epsilon_\theta)Q\ - Q^{\ 2}/2 - p\ , & \text{if } \theta+\epsilon_\theta \geq Q\ . & \text{(EC.111c)} \end{cases}$$

Therefore, the expected surplus $\overline{s}(p\ \ ,Q\ \ |\theta)$ of this type-θ subscriber is

$$\begin{aligned}
&\overline{s}(p\ \ ,Q\ \ |\theta)\\
=\ &\mathsf{E}_{\epsilon_\theta}[s\ \ (d\ \ |\theta+\epsilon_\theta)]\\
=\ &\mathsf{P}(\theta+\epsilon_\theta<0)\mathsf{E}_{\epsilon_\theta}[-p\ \ |\theta+\epsilon_\theta<0]+\mathsf{P}(0\le\theta+\epsilon_\theta<Q\ \ )\mathsf{E}_{\epsilon_\theta}\left[(\theta+\epsilon_\theta)^2/2-p\ \ |0\le\theta+\epsilon_\theta<Q\ \ \right]\\
&+\mathsf{P}(\theta+\epsilon_\theta\ge Q\ \ )\mathsf{E}_{\epsilon_\theta}\left[(\theta+\epsilon_\theta)Q\ \ -Q\ \ ^2/2-p\ \ |\theta+\epsilon_\theta\ge Q\ \ \right]\\
=\ &\mathsf{P}(0\le\theta+\epsilon_\theta<Q\ \ )\mathsf{E}_{\epsilon_\theta}\left[(\theta+\epsilon_\theta)^2/2\,|\,0\le\theta+\epsilon_\theta<Q\ \ \right]\\
&+\mathsf{P}(\theta+\epsilon_\theta\ge Q\ \ )\mathsf{E}_{\epsilon_\theta}\left[(\theta+\epsilon_\theta)Q\ \ -Q\ \ ^2/2\,|\,\theta+\epsilon_\theta\ge Q\ \ \right]-p\\
=\ &\mathsf{E}_{\epsilon_\theta}\left[(\theta+\epsilon_\theta)^2/2\right]-\mathsf{P}(\theta+\epsilon_\theta\ge Q\ \ )\mathsf{E}_{\epsilon_\theta}\left[(\theta+\epsilon_\theta-Q\ \ )^2/2\,|\,\theta+\epsilon_\theta\ge Q\ \ \right]-p\ \ .
\end{aligned}\tag{EC.112}$$

Note that (EC.112) implies that $\overline{s}(p\ \ ,Q\ \ |\theta)$ strictly increases in Q . Thus, increasing Q to $\theta+\epsilon_\theta^u$ while fixing p improves all subscribers' surpluses and they would still subscribe to the new contract. In the meanwhile, non-subscribers may also sign up for the new contract since p stays but Q increases. Therefore, the lemma holds.

Now we turn to demonstrate $\overline{\Pi}\ \ \le\overline{\Pi}_s$. Let $(p\ \ ,Q\ \ )$ denote the optimal bucket contract such that $Q\ \ \ge\theta+\epsilon_\theta^u$ for any subscriber of a given type θ. We shall prove that an bucket subscriber earns less utility for any valuation perturbation ϵ_θ than does she subscribe to the sharing contract $(p_s,Q_s)=(p\ \ ,Q\ \ )$. Therefore, any customer who subscribes to the bucket contract $(p\ \ ,Q\ \ )$ would also subscribe to the sharing contract $(p_s,Q_s)=(p\ \ ,Q\ \ )$.

Suppose that a type-θ customer subscribes to the optimal bucket contract $(p\ \ ,Q\ \ )$. For a realized ϵ_θ, (EC.111) characterizes her surplus. For the same type-θ subscriber, if she were offered a sharing contract $(p_s,Q_s)=(p\ \ ,Q\ \ )$, by (19) and (20) her surplus can be written as

$$s_s(d_s\,|\,\theta+\epsilon_\theta)=\begin{cases}-p_s+\hat{p}Q_s=-p\ \ +\hat{p}_sQ\ \ , & \text{if } \theta+\epsilon_\theta<\hat{p}_s\\ \dfrac{1}{2}(\theta+\epsilon_\theta-\hat{p}_s)^2-p_s+\hat{p}_sQ\ \ =\dfrac{1}{2}(\theta+\epsilon_\theta-\hat{p}_s)^2-p\ \ +\hat{p}_sQ\ \ , & \text{if } \theta+\epsilon_\theta\ge\hat{p}_s.\end{cases}$$

To show $s\ \ (d\ \ |\theta+\epsilon_\theta)\le s_s(d_s\,|\,\theta+\epsilon_\theta)$, let us consider the following three cases:

(i) $\theta+\epsilon_\theta<0$. In this case, $s(d\ \ |\theta+\epsilon_\theta)=-p\ \ \le-p\ \ +\hat{p}_sQ\ \ =s_s(d_s\,|\,\theta+\epsilon_\theta)$ since $\hat{p}_s\ge 0$.

(ii) $0\le\theta+\epsilon_\theta<\hat{p}_s$. In this case, $s\ \ (d\ \ |\theta+\epsilon_\theta)=\frac{1}{2}(\theta+\epsilon_\theta)^2-p\ \ <\hat{p}_sQ\ \ -p\ \ =s_s(d_s\,|\,\theta+\epsilon_\theta)$ since $(\theta+\epsilon_\theta)/2<\theta+\epsilon_\theta^u\le Q$ by Lemma EC3.

(iii) $\theta+\epsilon_\theta\ge\hat{p}_s$. In this case, $s\ \ (d\ \ |\theta+\epsilon_\theta)=\frac{1}{2}(\theta+\epsilon_\theta)^2-p\ \ \le\frac{1}{2}(\theta+\epsilon_\theta)^2+\hat{p}_s(Q\ \ -\theta-\epsilon_\theta)+\frac{1}{2}\hat{p}_s^2-p\ \ =\frac{1}{2}(\theta+\epsilon_\theta-\hat{p}_s)^2+\hat{p}_sQ\ \ -p\ \ =s_s(d_s\,|\,\theta+\epsilon_\theta)$ since $\hat{p}_s\ge 0$ and $\theta+\epsilon_\theta\le\theta+\epsilon_\theta^u\le Q$ by Lemma EC3.

Therefore, $\overline{\Pi}\ \ \le\overline{\Pi}_s$.

Technical Supplement of "Digital Goods Reselling: Implications on Cannibalization and Price Discrimination"

TS1. Ancillary Results of Proposition 1

LEMMA TS1. *Assume that the provider offers a sharing contract* (p_s, Q_s, t_s). *There exists a unique* $\bar{\theta}_s$ *such that customers subscribe to the service if and only if* $\theta \geq \bar{\theta}_s$. *The subscriber's demand satisfies*

$$d_s(\theta) = \begin{cases} \max\{\theta - \hat{p}_s + t_s, 0\}, & \text{if } \bar{\theta}_s \leq \theta < \hat{p}_s + Q_s - t_s \\ Q_s, & \text{if } \hat{p}_s + Q_s - t_s \leq \theta \leq \hat{p}_s + Q_s + t_s \\ \theta - \hat{p}_s - t_s, & \text{otherwise} \end{cases} \tag{TS.1}$$

where $\hat{p}_s$ *is market-clearing price of a unit of the goods.*

Proof of Lemma TS1. If a customer decides to subscribe to the service, i.e., $s_s(d_s \mid \theta) \geq 0$, we can derive her marginal utility change

$$\frac{\partial s_s}{\partial d_s} = \begin{cases} \theta - d_s - \hat{p}_s + t_s, & \text{if } 0 \leq d_s \leq Q_s \\ \theta - d_s - \hat{p}_s - t_s, & \text{otherwise .} \end{cases}$$

First, there must be a unique optimal d_s because the marginal utility change is monotone in d_s. Moreover, since $d_s \geq 0$, we have that if $\theta < \hat{p}_s - t_s$, then $\frac{\partial s_s}{\partial d_s} < 0$ and $d_s = 0$; If $\hat{p}_s - t_s \leq \theta < \hat{p}_s + Q_s - t_s$, $s_s(d_s \mid \theta)$ is maximized at $d_s(\theta) = \theta - \hat{p}_s + t_s \leq Q_s$; If $\hat{p}_s + Q_s - t_s \leq \theta \leq \hat{p}_s + Q_s + t_s$, for $d_s \leq Q_s$, $\frac{\partial s_s}{\partial d_s} = \theta - d_s - \hat{p}_s + t_s \geq 0$, for $d_s \geq Q_s$, $\frac{\partial s_s}{\partial d_s} = \theta - d_s - \hat{p}_s - t_s \leq 0$, then $s_s(d_s \mid \theta)$ is maximized at $d_s(\theta) = Q_s$; Otherwise, $s_s(d_s \mid \theta)$ is maximized at $d_s(\theta) = \theta - \hat{p}_s - t_s \geq Q_s$. By (4),

$$d_s(\theta) = \begin{cases} 0, & \text{if } 0 \leq \theta < \hat{p}_s - t_s \\ \theta - \hat{p}_s + t_s, & \text{if } \hat{p}_s - t_s \leq \theta < \hat{p}_s + Q_s - t_s \\ Q_s, & \text{if } \hat{p}_s + Q_s - t_s \leq \theta \leq \hat{p}_s + Q_s + t_s \\ \theta - \hat{p}_s - t_s, & \text{otherwise} \end{cases}$$

and

$$s_s\left(d_s(\theta) \mid \theta\right) = \begin{cases} (\hat{p}_s - t_s)Q_s - p_s, & \text{if } 0 \leq \theta < \hat{p}_s - t_s \\ \frac{1}{2}(\theta - \hat{p}_s + t_s)^2 + (\hat{p}_s - t_s)Q_s - p_s, & \text{if } \hat{p}_s - t_s \leq \theta < \hat{p}_s + Q_s - t_s \\ \theta Q_s - \frac{1}{2}Q_s^2 - p_s, & \text{if } \hat{p}_s + Q_s - t_s \leq \theta \leq \hat{p}_s + Q_s + t_s \\ \frac{1}{2}(\theta - \hat{p}_s - t_s)^2 + (\hat{p}_s + t_s)Q_s - p_s, & \text{otherwise.} \end{cases}$$

If $(\hat{p}_s - t_s)Q_s - p_s \geq 0$, customers of type $\theta < \hat{p}_s - t_s$ are speculators who have the intention of purchasing for resales only (without any self-consumption) and customers of type $\theta \geq \hat{p}_s - t_s$ purchase with strictly positive consumption. Note that not all customers of type $\theta < \hat{p}_s - t_s$ would subscribe since the market clearing price $\hat{p}_s$ is endogenous. Thus, only a fraction of customers of type $\theta < \hat{p}_s - t_s$ eventually subscribe. With loss of generality, we define $\bar{\theta}_s$ as the largest θ that is smaller than $\hat{p}_s - t_s$ and yields the market clearing price $\hat{p}_s$. (Who are the subscribing speculators is not critical as long as the measure of the subscribing speculator set is fixed. Moreover, we will show that the provider will price out speculators at optimality.)

If $(\hat{p}_s - t_s)Q_s - p_s \geq 0$, $s_s(d_s(\theta) \mid \theta)$ strictly increases in θ for $\theta \geq \hat{p}_s - t_s$. Hence, there exists a unique $\bar{\theta}_s \geq \hat{p}_s - t_s$ such that $s_s(d_s(\theta) \mid \theta) \geq 0$ if and only if $\theta \geq \bar{\theta}_s$. In particular, let $\bar{\theta}_s \geq 0$ be the solution to

$$s_s\left(d_s(\theta)|\theta\right) = \frac{1}{2}(\theta - \hat{p}_s + t_s)^2 + (\hat{p}_s - t_s)Q_s - p_s = 0. \tag{TS.2}$$

We thus claim that customers subscribe to the service if and only if $\theta \geq \bar{\theta}_s$.

LEMMA TS2. *Assume that the service provider offers a sharing contract (p_s, Q_s, t_s). Let $\bar{\theta}_s$ be the cutoff such that customers of type $\theta \geq \bar{\theta}_s$ subscribe and let $\hat{p}_s$ be the equilibrium market clearing price.*

(i) The sharing market has an equilibrium with speculators, i.e., some subscribers resell all their allowances, if and only if $\hat{p}_s \geq p_s/Q_s + t_s$ and $0 \leq \bar{\theta}_s < \hat{p}_s - t_s$. Moreover, $\hat{p}_s$ and $\bar{\theta}_s$ must satisfy the following equality

$$\begin{aligned} Q_s\left[\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s-t_s} f(\theta)\,d\theta + \int_{\hat{p}_s+Q_s+t_s}^{\Theta} f(\theta)\,d\theta\right] + \hat{p}_s\left[\int_{\hat{p}_s-t_s}^{\hat{p}_s+Q_s-t_s} f(\theta)\,d\theta + \int_{\hat{p}_s+Q_s+t_s}^{\Theta} f(\theta)\,d\theta\right] \\ = \int_{\hat{p}_s-t_s}^{\hat{p}_s+Q_s-t_s} (\theta + t_s) f(\theta)\,d\theta + \int_{\hat{p}_s+Q_s+t_s}^{\Theta} (\theta - t_s) f(\theta)\,d\theta. \end{aligned} \tag{TS.3}$$

(ii) The sharing market has an equilibrium without speculators, i.e., all subscribers consume some of the allowance, if and only if $t_s \leq \hat{p}_s \leq p_s/Q_s + t_s$ and $\bar{\theta}_s \geq \hat{p}_s - t_s$. Moreover, $\hat{p}_s$ and $\bar{\theta}_s$ must satisfy the following equality

$$(Q_s + \hat{p}_s)\left[\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s-t_s} f(\theta)\,d\theta + \int_{\hat{p}_s+Q_s+t_s}^{\Theta} f(\theta)\,d\theta\right] = \int_{\bar{\theta}_s}^{\hat{p}_s+Q_s-t_s} (\theta + t_s) f(\theta)\,d\theta + \int_{\hat{p}_s+Q_s+t_s}^{\Theta} (\theta - t_s) f(\theta)\,d\theta. \tag{TS.4}$$

Proof of Lemma TS2. (i) By Lemma TS1, for speculators who does not consume any data, $d_s(\theta) = \max\{\theta - \hat{p}_s + t_s, 0\} = 0$, which occurs if and only if $\bar{\theta}_s < \hat{p}_s - t_s$. For these speculators, the fact that they subscribe to the service indicates that $s_s(d_s(\theta) \mid \theta) = (\hat{p}_s - t_s)Q_s - p_s \geq 0$. Thus, $\hat{p}_s \geq p_s/Q_s + t_s$. In this case, the total supply and demand of the sharing market are

$$\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s-t_s} [Q_s - d_s(\theta)] f(\theta)\mathrm{d}\theta = \int_{\bar{\theta}_s}^{\hat{p}_s-t_s} Q_s f(\theta)\mathrm{d}\theta + \int_{\hat{p}_s-t_s}^{\hat{p}_s+Q_s-t_s} \left(Q_s - (\theta - \hat{p}_s + t_s)\right) f(\theta)\mathrm{d}\theta \tag{TS.5}$$

and

$$\int_{\hat{p}_s+Q_s+t_s}^{\Theta} \left(d_s(\theta) - Q_s\right) f(\theta)\mathrm{d}\theta = \int_{\hat{p}_s+Q_s+t_s}^{\Theta} \left((\theta - \hat{p}_s - t_s) - Q_s\right) f(\theta)\mathrm{d}\theta, \tag{TS.6}$$

respectively. Equating (TS.5) and (TS.6), we attain the market clearing condition (TS.3).

(ii) If there are no speculators, Lemma TS1 indicates that $d_s(\theta) = \theta - \hat{p}_s + t_s$ for $\bar{\theta}_s \leq \theta < \hat{p}_s + Q_s - t_s$, which occurs if and only if $\bar{\theta}_s \geq \hat{p}_s - t_s$. For these subscribers, $s_s(d_s(\theta) \mid \theta) \geq 0$. In particular, for the subscriber of type $\bar{\theta}_s$, $s_s(d_s(\bar{\theta}_s) \mid \bar{\theta}_s) = \frac{1}{2}(\bar{\theta}_s - \hat{p}_s + t_s)^2 - p_s + (\hat{p}_s - t_s)Q_s = 0$, or equivalently, $\frac{1}{2}(\bar{\theta}_s - \hat{p}_s + t_s)^2 = p_s - (\hat{p}_s - t_s)Q_s \geq 0$. Thus, $\hat{p}_s \leq p_s/Q_s + t_s$. In this case, the total supply and demand of the sharing market are

$$\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s-t_s} \left(Q_s - d_s(\theta)\right) f(\theta)\mathrm{d}\theta = \int_{\bar{\theta}_s}^{\hat{p}_s+Q_s-t_s} \left(Q_s - (\theta - \hat{p}_s + t_s)\right) f(\theta)\mathrm{d}\theta \tag{TS.7}$$

and

$$\int_{\hat{p}_s+Q_s+t_s}^{\Theta} \left(d_s(\theta) - Q_s\right) f(\theta)\mathrm{d}\theta = \int_{\hat{p}_s+Q_s+t_s}^{\Theta} \left((\theta - \hat{p}_s - t_s) - Q_s\right) f(\theta)\mathrm{d}\theta, \tag{TS.8}$$

respectively. Equating (TS.7) and (TS.8) for market clearing, we obtain in (TS.4).

TS2. Ancillary Results of Theorem 3

Lemma TS3. *Assume that the service provider offers a K-tier menu of sharing contracts, denoted by $\{p_{s_k}, Q_{s_k}\}$, $k = 1, 2 \ldots, K$, with ascending market clearing prices $\hat{p}_{s_1} \le \hat{p}_{s_2} \le \ldots \le \hat{p}_{s_K}$. Then,*

(i) for any given two items $i < j$, $s_{s_i}(d_{s_i}(\theta) \mid \theta) - s_{s_j}(d_{s_j}(\theta) \mid \theta)$ (weakly) increases in θ;

(ii) if customers of types $\underline{\theta}$ and $\tilde{\theta}$ choose item s_k of the menu, so do all customers of types $\theta \quad [\underline{\theta}, \tilde{\theta}]$.

Proof of Lemma TS3. (i) By Eq. (TS.2) and Lemma 1, if $\theta \le \hat{p}_{s_i}$, then $s_{s_i}(d_{s_i}(\theta) \mid \theta) - s_{s_j}(d_{s_j}(\theta) \mid \theta) = (-p_{s_i} + \hat{p}_{s_i} Q_{s_i}) - (-p_{s_j} + \hat{p}_{s_j} Q_{s_j})$ is constant; if $\hat{p}_{s_i} \le \theta \le \hat{p}_{s_j}$, then $s_{s_i}(d_{s_i}(\theta) \mid \theta) - s_{s_j}(d_{s_j}(\theta) \mid \theta) = \frac{1}{2}(\theta - \hat{p}_{s_i})^2 - p_{s_i} + \hat{p}_{s_i} Q_{s_i} - (-p_{s_j} + \hat{p}_{s_j} Q_{s_j})$ increases in θ; if $\hat{p}_{s_j} \le \theta$, then $s_{s_i}(d_{s_i}(\theta) \mid \theta) - s_{s_j}(d_{s_j}(\theta) \mid \theta) = \frac{1}{2}(\theta - \hat{p}_{s_i})^2 - p_{s_i} + \hat{p}_{s_i} Q_{s_i} - (\frac{1}{2}(\theta - \hat{p}_{s_j})^2 - p_{s_j} + \hat{p}_{s_j} Q_{s_j}) = \frac{1}{2}(\hat{p}_{s_j} - \hat{p}_{s_i})(2\theta - \hat{p}_{s_j} - \hat{p}_{s_i}) - p_{s_i} + \hat{p}_{s_i} Q_{s_i} - (-p_{s_j} + \hat{p}_{s_j} Q_{s_j})$ increases in θ. In sum, $s_{s_i}(d_{s_i}(\theta) \mid \theta) - s_{s_j}(d_{s_j}(\theta) \mid \theta)$ increases in θ.

(ii) We show the result by contradiction. Assume that there exists a type-θ customer who chooses a different item s_j than s_k, where $\theta \quad [\underline{\theta}, \tilde{\theta}]$. Without loss of generality, let us further assume $\hat{p}_{s_k} \le \hat{p}_{s_j}$ and consider $s_{s_k}(d_{s_k}(\theta) \mid \theta) - s_{s_j}(d_{s_j}(\theta) \mid \theta)$. The fact that both type-$\underline{\theta}$ and type-$\tilde{\theta}$ customers choose item s_k implies that $s_{s_k}(d_{s_k}(\underline{\theta}) \mid \underline{\theta}) - s_{s_j}(d_{s_j}(\underline{\theta}) \mid \underline{\theta}) \ge 0$ and $s_{s_k}(d_{s_k}(\tilde{\theta}) \mid \tilde{\theta}) - s_{s_j}(d_{s_j}(\tilde{\theta}) \mid \tilde{\theta}) \ge 0$, whereas for the type-θ customer $s_{s_k}(d_{s_k}(\theta) \mid \theta) - s_{s_j}(d_{s_j}(\theta) \mid \theta) \le 0$. Thus, $s_{s_k}(d_{s_k}(\theta) \mid \theta) - s_{s_j}(d_{s_j}(\theta) \mid \theta)$ is not monotone in θ, which contradicts with Lemma TS3(i).

Lemma TS4. *It is optimal to offer a K-tier menu of sharing contracts such that no speculators subscribe in equilibrium.*

Proof of Lemma TS4. Lemma TS3 implies that a K-tier menu will separate customers to K intervals according to their types. Without of loss generality, assume that customers in the i-th interval choose item s_i. Moreover, let the left endpoint of the i-th interval denote as $\bar{\theta}_i$ and $\bar{\theta}_1 \le \bar{\theta}_2 \le \ldots, \le \bar{\theta}_K$.

We prove the result by contradiction. Suppose that there exist some speculators who choose item s_i. By Lemma 2(i), we have $\bar{\theta}_i < \hat{p}_{s_i}$ and $\hat{p}_{s_i} Q_{s_i} - p_{s_i} \ge 0$.

First, we will show that speculators must be among those who choose s_1. Assume that speculators choose an item s_i where $i > 1$. It can be shown that a type-$\bar{\theta}_i$ subscriber must be a speculator and $s_{s_{i-1}}(d_{s_i}(\bar{\theta}_i) \mid \bar{\theta}_i) = s_{s_i}(d_{s_i}(\bar{\theta}_i) \mid \bar{\theta}_i) = \hat{p}_{s_i} Q_{s_i} - p_{s_i} > 0$ at optimality. Then, there must exist another speculator of type $\theta = \bar{\theta}_i + \epsilon \quad (\bar{\theta}_i, \hat{p}_{s_i})$ purchasing item s_i as well, i.e., $s_{s_i}(d_{s_i}(\theta) \mid \theta) = \hat{p}_{s_i} Q_{s_i} - p_{s_i} > s_{s_{i-1}}(d_{s_i}(\theta) \mid \theta)$. However, for such a speculator, we also have $s_{s_{i-1}}(d_{s_i}(\theta) \mid \theta) \ge s_{s_{i-1}}(d_{s_i}(\bar{\theta}_i) \mid \bar{\theta}_i) = s_{s_i}(d_{s_i}(\bar{\theta}_i) \mid \bar{\theta}_i) = s_{s_i}(d_{s_i}(\theta) \mid \theta)$, where the first inequality is because $s_{s_{i-1}}(d_{s_i}(\bar{\theta}_i) \mid \bar{\theta}_i)$ increases in θ. A contradiction arises. Hence, if there exist any speculators, they must choose item s_1.

Next, we demonstrate that the optimal menu must eliminate all speculators from item s_1. By Lemma 2(i), if there are speculators purchasing s_1, then $\bar{\theta}_1 < \hat{p}_{s_1}$ and $\hat{p}_{s_1} Q_{s_1} - p_{s_1} \ge 0$.

Assume $\hat{p}_{s_1} Q_{s_1} - p_{s_1} > 0$ at optimality. Then we can construct a new menu by setting $p_{s_1} = \hat{p}_{s_1} Q_{s_1}$ and $p_{s_i} = p_{s_i} + p_{s_1} - p_{s_1}$ for $i \ge 2$. It is easy to show that the surplus difference of any arbitrary two items of the menus is unchanged under this new menu $\{p_{s_i}, Q_{s_i}\}$, $i = 1, 2 \ldots, K$. Hence, customers who choose item s_i from menu $\{p_{s_i}, Q_{s_i}\}$ will still purchase item s_i from $\{p_{s_i}, Q_{s_i}\}$. Moreover, since

$p_{s_i} > p_{s_i}$ for any $i = 1, \ldots, K$, menu $\{p_{s_i}, Q_{s_i}\}$ thus induces a higher revenue, which contradicts the optimality of menu $\{p_{s_i}, Q_{s_i}\}$. Therefore, it is only possible that $\hat{p}_{s_1} Q_{s_1} - p_{s_1} = 0$ at optimality.

We then consider the case $\hat{p}_{s_1} Q_{s_1} - p_{s_1} = 0$ and show that there exists a menu $\{p_{s_i}, Q_{s_i}\}$ that induces no speculators but yields the same revenue. Specifically, for item s_1 let $Q_{s_1} = \int_{p_{s_1}/Q_{s_1}}^{\bar{\theta}_2} (\theta - p_{s_1}/Q_{s_1}) f(\theta) \mathrm{d}\theta / (\bar{F}(p_{s_1}/Q_{s_1}) - \bar{F}(\bar{\theta}_2))$ and choose p_{s_1} such that $p_{s_1}/Q_{s_1} = p_{s_1}/Q_{s_1}$. For item s_i, $i = 2, \ldots, K$, let $(p_{s_i}, Q_{s_i}) = (p_{s_i}, Q_{s_i})$. Note that the market clearing equation (6) is achieved at $\bar{\theta}_s = \hat{p}_s = p_{s_1}/Q_{s_1} = \hat{p}_{s_1}$ under (p_{s_1}, Q_{s_1}) (with Θ in (6) being replaced with $\bar{\theta}_2$). Thus, by Lemma 2(ii), sharing without speculators occurs under $\{p_{s_1}, Q_{s_1}\}$. In other words, menu $\{p_{s_i}, Q_{s_i}\}$ does not induce any speculators. Next, we prove that $\{p_{s_i}, Q_{s_i}\}$ yields the same revenue as $\{p_{s_i}, Q_{s_i}\}$. For any type θ subscriber, $s_{s_1}\left(d_{s_1}(\theta) | \theta\right) = \frac{1}{2}(\theta - \hat{p}_{s_1})^2 + \hat{p}_{s_1} Q_{s_1} - p_{s_1} = \frac{1}{2}(\theta - \hat{p}_{s_1})^2 + \hat{p}_{s_1} Q_{s_1} - p_{s_1}$ due to $p_{s_1}/Q_{s_1} = p_{s_1}/Q_{s_1} = \hat{p}_{s_1}$. This indicates that any customer earns the same surplus from (p_{s_1}, Q_{s_1}) as (p_{s_1}, Q_{s_1}). Thus, for $i \geq 2$, subscribers who choose item s_i under $\{p_{s_i}, Q_{s_i}\}$ will still purchase it under $\{p_{s_i}, Q_{s_i}\}$ and generate the same revenue for the provider. Subscribers of item s_1 also generate the same revenue under (p_{s_1}, Q_{s_1}) as (p_{s_1}, Q_{s_1}). To see this, recall that $p_{s_1}/Q_{s_1} = p_{s_1}/Q_{s_1}$ and $Q_{s_1}[\bar{F}(p_{s_1}/Q_{s_1}) - \bar{F}(\bar{\theta}_2)] = \int_{p_{s_1}/Q_{s_1}}^{\bar{\theta}_2} (\theta - p_{s_1}/Q_{s_1}) f(\theta) \mathrm{d}\theta$. Then, we have

$$\begin{aligned}
p_{s_1}[\bar{F}(\bar{\theta}_1) - \bar{F}(\bar{\theta}_2)] &= (p_{s_1}/Q_{s_1})[\bar{F}(p_{s_1}/Q_{s_1}) - \bar{F}(\bar{\theta}_2)] Q_{s_1} \\
&= (p_{s_1}/Q_{s_1})[\bar{F}(p_{s_1}/Q_{s_1}) - \bar{F}(\bar{\theta}_2)] Q_{s_1} \\
&= (p_{s_1}/Q_{s_1}) \int_{p_{s_1}/Q_{s_1}}^{\bar{\theta}_2} (\theta - p_{s_1}/Q_{s_1}) f(\theta) \mathrm{d}\theta \\
&= (p_{s_1}/Q_{s_1}) \int_{\hat{p}_{s_1}}^{\bar{\theta}_2} (\theta - \hat{p}_{s_1}) f(\theta) \mathrm{d}\theta \\
&= p_{s_1}[\bar{F}(\bar{\theta}_1) - \bar{F}(\bar{\theta}_2)],
\end{aligned}$$

where the second last equality is due to $\hat{p}_{s_1} = p_{s_1}/Q_{s_1}$ and the last equality is due to (5) which implies that $Q_{s_1}[\bar{F}(\bar{\theta}_1) - \bar{F}(\bar{\theta}_2)] = \int_{\hat{p}_{s_1}}^{\bar{\theta}_2} (\theta - \hat{p}_{s_1}) f(\theta) \mathrm{d}\theta$ (with Θ being replaced with $\bar{\theta}_2$).

Therefore, it is optimal to offer a K-tier menu such that no speculators exist in equilibrium.

Lemma TS5. *For any K-tier menu of three-part tariffs, denoted by $\{p_{n_k}, Q_{n_k}, \hat{p}_{n_k}\}$, $k = 1, 2 \ldots, K$, it is always possible to construct a revenue-equivalent counterpart such that if customers of types $\underline{\theta}$ and $\tilde{\theta}$ choose item n_k of the menu, so do all customers of types θ $[\underline{\theta}, \tilde{\theta}]$.*

Proof of Lemma TS5. By Proposition 2 of Bhargava and Gangwar (2018), a menu of K three-part tariffs, has a revenue-equivalent two-part tariffs if $\frac{\partial}{\partial \theta}\left(\frac{u(d|\theta)}{\frac{\partial u(d|\theta)}{\partial \theta}} \frac{f(\theta)}{\bar{F}(\theta)}\right) \geq 0$. In our case, $u(d \,|\, \theta) = \theta d - \frac{1}{2} d^2$, then $\frac{u(d|\theta)}{\frac{\partial u(d|\theta)}{\partial \theta}} \frac{f(\theta)}{\bar{F}(\theta)} = (\theta - \frac{1}{2} d) \frac{f(\theta)}{\bar{F}(\theta)}$ increases in $\theta \geq d$ since $F(\cdot)$ has an increasing failure rate. Hence, $\frac{\partial}{\partial \theta}\left(\frac{u(d|\theta)}{\frac{\partial u(d|\theta)}{\partial \theta}} \frac{f(\theta)}{\bar{F}(\theta)}\right) \geq 0$. Then, for any menu of a three-part tariff, we only need to show the result for its revenue-equivalent counterpart with $Q_{n_k} = 0$. Then, for any n_k, $d_{n_k}(\theta) = \theta - \hat{p}_{n_k}$ by (10). $s_{n_k}(d_{n_k}(\theta) \,|\, \theta) - s_{n_j}(d_{n_j}(\theta) \,|\, \theta) = \frac{1}{2}(\theta - \hat{p}_{n_k})^2 - p_{n_k} - (\frac{1}{2}(\theta - \hat{p}_{n_j})^2 - p_{n_j} = \frac{1}{2}(\hat{p}_{n_j} - \hat{p}_{n_k})(2\theta - \hat{p}_{n_j} - \hat{p}_{n_k}) - p_{n_k} + p_{n_j}$ is monotonous in θ. Following a similar procedure as the proof of Lemma TS3, we can show that if customers of types $\underline{\theta}$ and $\tilde{\theta}$ choose item n_k , all customers of types θ $[\underline{\theta}, \tilde{\theta}]$ do the same.

TS3. Subscription Decisions under Uncertainty

We first consider customers' subscription decisions under the bucket contract.

We start with customers' consumption decisions first. Assume that a type-θ customer with a valuation perturbation ϵ_θ subscribes to the bucket contract (p, Q). We can write her surplus as

$$s(d \mid \theta+\epsilon_\theta) = u(d \mid \theta+\epsilon_\theta) - c(d \mid \theta+\epsilon_\theta) = (\theta+\epsilon_\theta)d - \frac{1}{2}d^2 - p, \tag{TS.9}$$

from which the surplus-maximizing demand can be derived as

$$d\ \ (\theta+\epsilon_\theta) = \begin{cases} 0, & \text{if } \theta+\epsilon_\theta < 0 \\ \theta+\epsilon_\theta, & \text{if } 0 \leq \theta+\epsilon_\theta < Q \\ Q, & \text{if } \theta+\epsilon_\theta \geq Q. \end{cases}$$

Then, by (TS.9), a type-θ customer's maximum surplus under (p, Q) is

$$s(d\ \ \mid \theta+\epsilon_\theta) = u(d\ \ \mid \theta+\epsilon_\theta) - c(d\ \ \mid \theta+\epsilon_\theta) = \begin{cases} -p, & \text{if } \theta+\epsilon_\theta < 0 \\ (\theta+\epsilon_\theta)^2/2 - p, & \text{if } 0 \leq \theta+\epsilon_\theta < Q \\ (\theta+\epsilon_\theta)Q - Q^2/2 - p, & \text{if } \theta+\epsilon_\theta \geq Q. \end{cases}$$

Whether type-θ customers subscribe to the bucket contract (p, Q) is determined by their expected surplus. Specifically, type-θ customers subscribe to a bucket contract if and only if

$$\begin{aligned}
\overline{s}(p, Q \mid \theta) &= \mathsf{E}_{\epsilon_\theta}\left[s(d\ \ \mid \theta+\epsilon_\theta)\right] \\
&= \mathsf{P}(\theta+\epsilon_\theta < 0)\mathsf{E}_{\epsilon_\theta}\left[-p \mid \theta+\epsilon_\theta < 0\right] + \mathsf{P}(0 \leq \theta+\epsilon_\theta < Q)\mathsf{E}_{\epsilon_\theta}\left[(\theta+\epsilon_\theta)^2/2 - p \mid 0 \leq \theta+\epsilon_\theta < Q\right] \\
&\quad + \mathsf{P}(\theta+\epsilon_\theta \geq Q)\mathsf{E}_{\epsilon_\theta}\left[(\theta+\epsilon_\theta)Q - Q^2/2 - p \mid \theta+\epsilon_\theta \geq Q\right] \\
&= \mathsf{P}(0 \leq \theta+\epsilon_\theta < Q)\mathsf{E}_{\epsilon_\theta}\left[(\theta+\epsilon_\theta)^2/2 \mid 0 \leq \theta+\epsilon_\theta < Q\right] \\
&\quad + \mathsf{P}(\theta+\epsilon_\theta \geq Q)\mathsf{E}_{\epsilon_\theta}\left[(\theta+\epsilon_\theta)Q - Q\ ^2/2 \mid \theta+\epsilon_\theta \geq Q\right] - p \\
&= \mathsf{E}_{\epsilon_\theta}\left[(\theta+\epsilon_\theta)^2/2\right] - \mathsf{P}(\theta+\epsilon_\theta \geq Q)\mathsf{E}_{\epsilon_\theta}\left[(\theta+\epsilon_\theta - Q)^2/2 \mid \theta+\epsilon_\theta \geq Q\right] - p. \\
&\geq 0.
\end{aligned}$$

For ease of exposition, we thus denote $\Theta(p, Q) := \{\theta \mid \overline{s}(p, Q \mid \theta) \geq 0\}$ as the subscriber set under the bucket contract.

We then consider customers' subscription decisions under the nonlinear contract in this section.

We start with customers' consumption decisions first. Assume that a type-θ customer with a valuation perturbation ϵ_θ subscribes to the nonlinear contract. We can write her surplus as

$$s_n(d_n \mid \theta+\epsilon_\theta) = u(d_n \mid \theta+\epsilon_\theta) - c_n(d_n \mid \theta+\epsilon_\theta) = (\theta+\epsilon_\theta)d_n - \frac{1}{2}d_n^2 - \left(p_n + \hat{p}_n \cdot (d_n - Q_n)^+\right).$$

Similar to Lemma 3, we can derive the surplus-maximizing demand as

$$d_n(\theta+\epsilon_\theta) = \begin{cases} 0, & \text{if } \theta+\epsilon_\theta < 0 & \text{(TS.11a)} \\ \theta+\epsilon_\theta, & \text{if } 0 \leq \theta+\epsilon_\theta < Q_n & \text{(TS.11b)} \\ Q_n, & \text{if } Q_n \leq \theta+\epsilon_\theta < \hat{p}_n + Q_n & \text{(TS.11c)} \\ \theta+\epsilon_\theta - \hat{p}_n & \text{if } \hat{p}_n + Q_n \leq \theta+\epsilon_\theta. & \text{(TS.11d)} \end{cases}$$

Whether customers subscribe to a nonlinear contract are determined by their expected surplus under uncertainty. Specifically, type-θ customers subscribe to a nonlinear contract if and only if

$$\begin{aligned}
&\overline{s}_n(p_n, Q_n, \hat{p}_n \mid \theta) \\
= \ &\mathsf{E}_{\epsilon_\theta}\left[s_n(d_n \mid \theta+\epsilon_\theta)\right]
\end{aligned}$$

$$
\begin{aligned}
&= \mathsf{P}(\theta+\epsilon_\theta<0)\mathsf{E}_{\epsilon_\theta}\left[s_n(d_n)\mid\theta+\epsilon_\theta<0\right]+\mathsf{P}(0\le\theta+\epsilon_\theta<Q_n)\mathsf{E}_{\epsilon_\theta}\left[s_n(d_n)\mid 0\le\theta+\epsilon_\theta<Q_n\right]\\
&\quad+\mathsf{P}(Q_n\le\theta+\epsilon_\theta<\hat{p}_n+Q_n)\mathsf{E}_{\epsilon_\theta}\left[s_n(d_n)\mid Q_n\le\theta+\epsilon_\theta<\hat{p}_n+Q_n\right]\\
&\quad+\mathsf{P}(\hat{p}_n+Q_n\le\theta+\epsilon_\theta)\mathsf{E}_{\epsilon_\theta}\left[s_n(d_n)\mid\hat{p}_n+Q_n\le\theta+\epsilon_\theta\right]\\
&= \mathsf{P}(\theta+\epsilon_\theta<0)\mathsf{E}_{\epsilon_\theta}\left[-p_n\mid\theta+\epsilon_\theta<0\right]+\mathsf{P}(0\le\theta+\epsilon_\theta<Q_n)\mathsf{E}_{\epsilon_\theta}\left[\frac{1}{2}(\theta+\epsilon_\theta)^2-p_n\mid 0\le\theta+\epsilon_\theta<Q_n\right]\\
&\quad+\mathsf{P}(Q_n\le\theta+\epsilon_\theta<\hat{p}_n+Q_n)\mathsf{E}_{\epsilon_\theta}\left[(\theta+\epsilon_\theta)Q_n-\frac{1}{2}Q_n^2-p_n\mid Q_n\le\theta+\epsilon_\theta<\hat{p}_n+Q_n\right]\\
&\quad+\mathsf{P}(\hat{p}_n+Q_n\le\theta+\epsilon_\theta)\mathsf{E}_{\epsilon_\theta}\left[\frac{1}{2}(\theta+\epsilon_\theta-\hat{p}_n)^2+\hat{p}_nQ_n-p_n\mid\hat{p}_n+Q_n\le\theta+\epsilon_\theta\right]\\
&= \mathsf{P}(0\le\theta+\epsilon_\theta<Q_n)\mathsf{E}_{\epsilon_\theta}\left[\frac{1}{2}(\theta+\epsilon_\theta)^2\mid 0\le\theta+\epsilon_\theta<Q_n\right]\\
&\quad+\mathsf{P}(Q_n\le\theta+\epsilon_\theta<\hat{p}_n+Q_n)\mathsf{E}_{\epsilon_\theta}\left[\frac{1}{2}(\theta+\epsilon_\theta)Q_n-Q_n^2\mid Q_n\le\theta+\epsilon_\theta<\hat{p}_n+Q_n\right]\\
&\quad+\mathsf{P}(\hat{p}_n+Q_n\le\theta+\epsilon_\theta)\mathsf{E}_{\epsilon_\theta}\left[\frac{1}{2}(\theta+\epsilon_\theta-\hat{p}_n)^2+\hat{p}_nQ_n\mid\hat{p}_n+Q_n\le\theta+\epsilon_\theta\right]-p_n \qquad \text{(TS.12)}\\
&\ge 0.
\end{aligned}
$$

For ease of exposition, we thus denote $\Theta_n(p_n,Q_n,\hat{p}_n):=\{\theta\mid\bar{s}_n(p_n,Q_n,\hat{p}_n\mid\theta)\ge 0\}$ as the subscriber set under the nonlinear contract.

TS4. Ancillary Results of Section 6

LEMMA TS6. *Assume that the service provider offers a sharing contract (p_s,Q_s). There exists a unique $\bar{\theta}_s$ such that customers subscribe to the service if and only if $\theta\ge\bar{\theta}_s$. The subscriber's demand satisfies*

$$
d_s(\theta)=\begin{cases}0, & \text{if } 0\le\theta<\hat{p}_s-w_uQ_s\\ \frac{\theta-\hat{p}_s+w_uQ_s}{1+w_u}, & \text{if } \hat{p}_s-w_uQ_s\le\theta<\hat{p}_s+Q_s\\ \frac{\theta-\hat{p}_s+w_oQ_s}{1+w_o}, & \text{otherwise.}\end{cases} \qquad \text{(TS.13)}
$$

where $\hat{p}_s$ is the market clearing price of a unit of the good.

Proof of Lemma TS6. If a customer decides to subscribe to the service, i.e., $s_s(d_s\mid\theta)\ge 0$, we can derive her marginal utility change

$$
\frac{\partial s_s}{\partial d_s}=\begin{cases}\theta-(1+w_u)d_s-\hat{p}_s+w_uQ_s, & \text{if } d_s<Q_s \qquad \text{(TS.14)}\\ \theta-(1+w_o)d_s-\hat{p}_s+w_oQ_s, & \text{if } d_s\ge Q_s. \qquad \text{(TS.15)}\end{cases}
$$

First, there must be a unique optimal d_s because the marginal utility change is monotone in d_s.

We next write d_s as function of customer type θ. First, if $\theta<\hat{p}_s-w_uQ_s$, it is easy to see $\frac{\partial s_s}{\partial d_s}<0$ and $d_s=0$. Second, if $\hat{p}_s-w_uQ_s\le\theta<\hat{p}_s+Q_s$, it is to see that $d_s(\theta)=\frac{\theta-\hat{p}_s+w_uQ_s}{1+w_u}<Q_s$ by (TS.14). Third, if $\theta\ge\hat{p}_s+Q_s$, it is to see that $d_s(\theta)=\frac{\theta-\hat{p}_s+w_oQ_s}{1+w_o}\ge Q_s$ by (TS.15).

By (23),

$$
s_s(d_s(\theta)\mid\theta)=\begin{cases}-\frac{1}{2}w_uQ_s^2-p_s+\hat{p}_sQ_s, & \text{if } 0\le\theta<\hat{p}_s-w_uQ_s \qquad \text{(TS.16)}\\ \frac{(\theta-\hat{p}_s+w_uQ_s)^2}{2(1+w_u)}-\frac{1}{2}w_uQ_s^2-p_s+\hat{p}_sQ_s, & \text{if } \hat{p}_s-w_uQ_s\le\theta<\hat{p}_s+Q_s \qquad \text{(TS.17)}\\ \frac{(\theta-\hat{p}_s+w_oQ_s)^2}{2(1+w_o)}-\frac{1}{2}w_oQ_s^2-p_s+\hat{p}_sQ_s, & \text{if } \hat{p}_s+Q_s\le\theta\le\Theta.\end{cases}
$$

which shows that $s_s(d_s(\theta)\mid\theta)$ increases in θ. Hence, there exists a unique $\bar{\theta}_s\ge 0$ such that $s_s(d_s(\theta)\mid\theta)\ge 0$ if and only if $\theta\ge\bar{\theta}_s$.

LEMMA TS7. *Assume that the service provider offers a sharing contract* (p_s, Q_s). *Let* $\bar{\theta}_s$ *represent the cutoff so that customers of type* $\theta \geq \bar{\theta}_s$ *subscribe and* $\hat{p}_s$ *be the equilibrium market clearing price.*

(i) The sharing market has an equilibrium with speculators; i.e., some subscribers resell all their allowances at a unique market clearing price $\hat{p}_s$ *if and only if* $\hat{p}_s \geq p_s/Q_s + w_u Q_s/2$ *and* $0 \leq \bar{\theta}_s < \hat{p}_s - w_u Q_s$. *Moreover,* $\hat{p}_s$ *and* $\bar{\theta}_s$ *must satisfy the following equality*

$$Q_s \bar{F}(\bar{\theta}_s) = \int_{\hat{p}_s - w_u Q_s}^{\hat{p}_s + Q_s} \frac{\theta - \hat{p}_s + w_u Q_s}{1 + w_u} f(\theta)\, d\theta + \int_{\hat{p}_s + Q_s}^{\Theta} \frac{\theta - \hat{p}_s + w_o Q_s}{1 + w_o} f(\theta)\, d\theta. \tag{TS.18}$$

(ii) The sharing market has an equilibrium without speculators; i.e., all subscribers consume some of the allowance and there is a unique market clearing price $\hat{p}_s$, *if and only if* $\hat{p}_s \leq p_s/Q_s + w_u Q_s/2$ *and* $\bar{\theta}_s \geq \hat{p}_s - w_u Q_s$. *Moreover,* $\hat{p}_s$ *and* $\bar{\theta}_s$ *must satisfy the following equality*

$$Q_s \bar{F}(\bar{\theta}_s) = \int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} \frac{\theta - \hat{p}_s + w_u Q_s}{1 + w_u} f(\theta)\, d\theta + \int_{\hat{p}_s + Q_s}^{\Theta} \frac{\theta - \hat{p}_s + w_o Q_s}{1 + w_o} f(\theta)\, d\theta. \tag{TS.19}$$

Proof of Lemma TS7. (i) By Lemma TS6, for speculators who does not consume any data, $d_s(\theta) = 0$, which occurs if and only if $\bar{\theta}_s < \hat{p}_s - w_u Q_s$. For these speculators, the fact that they subscribe to the service indicates that $s_s(d_s(\theta) \mid \theta) = -\frac{1}{2} w_u Q_s^2 - p_s + \hat{p}_s Q_s \geq 0$ by (TS.16). Thus, $\hat{p}_s \geq p_s/Q_s + w_u Q_s/2$. In this case, the total supply and demand of the sharing market are

$$\int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} (Q_s - d_s(\theta)) f(\theta) \mathrm{d}\theta = \int_{\bar{\theta}_s}^{\hat{p}_s - w_u Q_s} Q_s f(\theta) \mathrm{d}\theta + \int_{\hat{p}_s - w_u Q_s}^{\hat{p}_s + Q_s} \left(Q_s - \frac{\theta - \hat{p}_s + w_u Q_s}{1 + w_u} \right) f(\theta) \mathrm{d}\theta \tag{TS.20}$$

and

$$\int_{\hat{p}_s + Q_s}^{\Theta} (d_s(\theta) - Q_s)\, f(\theta) \mathrm{d}\theta = \int_{\hat{p}_s + Q_s}^{\Theta} \left(\frac{\theta - \hat{p}_s + w_o Q_s}{1 + w_o} - Q_s \right) f(\theta) \mathrm{d}\theta, \tag{TS.21}$$

respectively. Equating (TS.20) and (TS.21) to attain the market clearing condition, we obtain (TS.18).

(ii) By Lemma TS6, the fact that there are no speculators means that $d_s(\theta) \geq 0$ for all subscribers, which occurs if and only if $\bar{\theta}_s \geq \hat{p}_s - w_u Q_s$. For these subscribers, $s_s(d_s(\theta) \mid \theta) \geq 0$. In particular, by (TS.17), for the subscriber of type $\bar{\theta}_s$,

$$s_s(d_s(\bar{\theta}_s) \mid \bar{\theta}_s) = \frac{(\bar{\theta}_s - \hat{p}_s + w_u Q_s)^2}{2(1 + w_u)} - \frac{1}{2} w_u Q_s^2 - p_s + \hat{p}_s Q_s = 0$$

or equivalently

$$\frac{(\bar{\theta}_s - \hat{p}_s + w_u Q_s)^2}{2(1 + w_u)} = \frac{1}{2} w_u Q_s^2 + p_s - \hat{p}_s Q_s \geq 0.$$

Thus, $\hat{p}_s \leq p_s/Q_s + w_u Q_s/2$. In this case, the total supply and demand of the sharing market are

$$\int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} (Q_s - d_s(\theta))\, f(\theta) \mathrm{d}\theta = \int_{\bar{\theta}_s}^{\hat{p}_s + Q_s} \left(Q_s - \frac{\theta - \hat{p}_s + w_u Q_s}{1 + w_u} \right) f(\theta) \mathrm{d}\theta \tag{TS.22}$$

and

$$\int_{\hat{p}_s + Q_s}^{\Theta} (d_s(\theta) - Q_s)\, f(\theta) \mathrm{d}\theta = \int_{\hat{p}_s + Q_s}^{\Theta} \left(\frac{\theta - \hat{p}_s + w_o Q_s}{1 + w_o} - Q_s \right) f(\theta) \mathrm{d}\theta, \tag{TS.23}$$

respectively. Equating (TS.22) and (TS.23) to attain the market clearing condition, we obtain (TS.19).

PROPOSITION TS1. *Assume that the service provider offers a sharing contract* (p_s, Q_s).

(i) The sharing market has a unique equilibrium with speculators if and only if $0 \leq Q_s < \overline{Q}_s(p_s, Q_s)$;

(ii) The sharing market has a unique equilibrium without speculators if and only if $Q_s \geq \overline{Q}_s(p_s, Q_s)$ *and* $p_s/Q_s + Q_s/2 \leq \Theta$,

where $\overline{Q}_s(p_s, Q_s) = \dfrac{\int_{\frac{p_s}{Q_s}-\frac{w_uQ_s}{2}}^{\frac{p_s}{Q_s}+\frac{w_uQ_s}{2}+Q_s} \frac{\theta - p_s/Q_s + w_uQ_s/2}{1+w_u} f(\theta)\, d\theta + \int_{\frac{p_s}{Q_s}+\frac{w_uQ_s}{2}+Q_s}^{\Theta} \frac{\theta - p_s/Q_s - w_uQ_s/2 + w_oQ_s}{1+w_o} f(\theta)\, d\theta}{\bar{F}\left(\frac{p_s}{Q_s} - \frac{w_uQ_s}{2}\right)}$.

Proof of Proposition TS1. For ease of exposition, we define $g(y) = \int_{y-w_uQ_s}^{y+Q_s} \frac{\theta - y + w_uQ_s}{1+w_u} f(\theta)\mathrm{d}\theta + \int_{y+Q_s}^{\Theta} \frac{\theta - y + w_oQ_s}{1+w_o} f(\theta)\mathrm{d}\theta$. Note that $g(y)$ strictly decreases in $y \geq w_uQ_s$. Moreover, $0 \leq g(y) \leq \int_{y-w_uQ_s}^{y+Q_s} \theta f(\theta)\mathrm{d}\theta + \int_{y+Q_s}^{\Theta} \frac{\theta - y + w_oQ_s}{1+w_o} f(\theta)\mathrm{d}\theta \leq \int_{y-w_uQ_s}^{y+Q_s} \theta f(\theta)\mathrm{d}\theta + \int_{y+Q_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta \leq \int_0^{y+Q_s} \theta f(\theta)\mathrm{d}\theta + \int_{y+Q_s}^{\Theta} \theta f(\theta)\mathrm{d}\theta = \mathsf{E}[\theta]$, where the second inequality is due to $-y + w_uQ_s \leq 0 \leq w_o\theta$, the third inequality is due to $-y + w_oQ_s \leq w_o(y + Q_s)$, and last third inequality is due to $y \geq w_uQ_s$.

(i) *Necessity.* Suppose that the sharing market has an unique equilibrium with speculators. By Lemma TS7(i), we have $0 \leq \bar{\theta}_s < \hat{p}_s - w_uQ_s$ and $\hat{p}_s \geq p_s/Q_s + w_uQ_s/2$. Consider two cases: $\bar{\theta}_s = 0$ and $\bar{\theta}_s > 0$.

If $\bar{\theta}_s = 0$,

$$\begin{aligned} Q_s =& Q_s\bar{F}(\bar{\theta}_s) = \int_{\hat{p}_s - w_uQ_s}^{\hat{p}_s + Q_s} \frac{\theta - \hat{p}_s + w_uQ_s}{1+w_u} f(\theta)\mathrm{d}\theta + \int_{\hat{p}_s + Q_s}^{\Theta} \frac{\theta - \hat{p}_s + w_oQ_s}{1+w_o} f(\theta)\mathrm{d}\theta \\ \leq& \overline{Q}_s(p_s, Q_s)\bar{F}(p_s/Q_s - w_uQ_s/2) < \overline{Q}_s(p_s, Q_s), \end{aligned}$$

where the first inequality is due to $g(y)$ strictly decreases in $y \geq w_uQ_s$ and $\hat{p}_s \geq p_s/Q_s + w_uQ_s/2$.

If $\bar{\theta}_s > 0$, we first prove $\hat{p}_s = p_s/Q_s + w_uQ_s/2$ by contradiction. Suppose $\hat{p}_s > p_s/Q_s + w_uQ_s/2$. For customers of type $\theta < \bar{\theta}_s$, if they subscribe, $d_s(\theta) = 0$ and $s_s(d_s(\theta) \mid \theta) = -\frac{1}{2}w_uQ_s^2 - p_s + \hat{p}_sQ_s > 0$, which contradicts with the fact that only customers with $\theta \geq \bar{\theta}_s$ subscribe. Since $0 \leq \bar{\theta}_s < \hat{p}_s - w_uQ_s$, then $Q_s\bar{F}(\bar{\theta}_s) = \int_{\hat{p}_s - w_uQ_s}^{\hat{p}_s + Q_s} \frac{\theta - \hat{p}_s + w_uQ_s}{1+w_u} f(\theta)\mathrm{d}\theta + \int_{\hat{p}_s + Q_s}^{\Theta} \frac{\theta - \hat{p}_s + w_oQ_s}{1+w_o} f(\theta)\mathrm{d}\theta > Q_s\bar{F}(\hat{p}_s - w_uQ_s)$ due to the monotonicity of $\bar{F}(\cdot)$. Note $\hat{p}_s = p_s/Q_s - w_uQ_s$, we have

$$\int_{\hat{p}_s - w_uQ_s}^{\hat{p}_s + Q_s} \frac{\theta - \hat{p}_s + w_uQ_s}{1+w_u} f(\theta)\mathrm{d}\theta + \int_{\hat{p}_s + Q_s}^{\Theta} \frac{\theta - \hat{p}_s + w_oQ_s}{1+w_o} f(\theta)\mathrm{d}\theta > Q_s\bar{F}(\hat{p}_s - w_uQ_s) \qquad Q_s < \overline{Q}_s.$$

Sufficiency. Suppose $0 \leq Q_s < \overline{Q}_s$. To ensure the existence of a sharing equilibrium with speculators, we need to show that (TS.18) has a unique solution $\hat{p}_s \geq 0$ and there exists a unique $0 \leq \bar{\theta}_s < \hat{p}_s - w_uQ_s$ such that $s_s(d_s(\bar{\theta}_s) \mid \bar{\theta}_s) \geq 0$, where the equality is achieved if $\bar{\theta}_s = 0$. Let us consider two cases: (a) $0 \leq Q_s < g(p_s/Q_s + w_uQ_s/2)$, and (b) $g(p_s/Q_s + w_uQ_s/2) \leq Q_s < \overline{Q}_s$.

(a) If $0 \leq Q_s \leq g(p_s/Q_s + w_uQ_s/2)$, we first prove $\bar{\theta}_s > 0$ does not occur in equilibrium. Then, we construct a sharing with speculators equilibrium with $\bar{\theta}_s = 0$ and show that this is the only possible equilibrium.

Suppose $\bar{\theta}_s > 0$ in equilibrium. From the proof of necessity, we know $\hat{p}_s = p_s/Q_s + w_uQ_s/2$ if $\bar{\theta}_s > 0$. Then,

$$Q_s\bar{F}(\bar{\theta}_s) < Q_s \leq g(p_s/Q_s + w_uQ_s/2) = g(\hat{p}_s),$$

which implies (TS.18) has no solution. Hence, it is not possible to have $\bar{\theta}_s > 0$ in equilibrium.

Next let $\bar{\theta}_s = 0$ and we show that there exists a unique $\hat{p}_s > p_s/Q_s + w_uQ_s/2$ such that the market-clearing condition (TS.18) holds. Note that $Q_s\bar{F}(\bar{\theta}_s = 0) = Q_s$. Hence, we can rewrite (TS.18) as

$Q_s = g(\hat{p}_s)$. Since $g(\hat{p}_s)$ $[0, \mathsf{E}[\theta]]$ for $\hat{p}_s \geq w_u Q_s$, there must exists a $\hat{p}_s$ such that $Q_s = g(\hat{p}_s)$ has a solution for a given $0 \leq Q_s < g(p_s/Q_s + w_u Q_s/2) \leq \mathsf{E}[\theta]$. Moreover, the strict monotonicity implies that such a $\hat{p}_s$ must be unique and $\hat{p}_s > p_s/Q_s + w_u Q_s/2$ since $Q_s = g(\hat{p}_s) < g(p_s/Q_s + w_u Q_s/2)$.

At last, we show that $s_s(d_s(\bar{\theta}_s) \mid \bar{\theta}_s) > 0$. Since $\bar{\theta}_s = 0 < p_s/Q_s + w_u Q_s/2 < \hat{p}_s$, $d_s(\bar{\theta}_s = 0) = 0$ By Lemma TS6. Hence, $s_s(d_s(\bar{\theta}_s = 0) \mid \bar{\theta}_s = 0) = -\frac{1}{2} w_u Q_s^2 - p_s + \hat{p}_s Q_s > 0$ by (TS.16).

(b) $g(p_s/Q_s + w_u Q_s/2) < Q_s < \overline{Q}_s$. We first prove that $\bar{\theta}_s = 0$ when sharing with speculators emerges in equilibrium. Suppose $\bar{\theta}_s = 0$. This means type-$\bar{\theta}_s$ customers do not value the service at all. They, thus, consume nothing even though they subscribe to it. Hence, $d_s(\bar{\theta}_s) = 0$ and $s_s(d_s(\bar{\theta}_s) \mid \bar{\theta}_s) = -\frac{1}{2} w_u Q_s^2 - p_s + \hat{p}_s Q_s \geq 0$, which implies $\hat{p}_s \geq p_s/Q_s + w_u Q_s/2$. Then, we have

$$Q_s \bar{F}(\bar{\theta}_s) = Q_s > g(p_s/Q_s + w_u Q_s/2) \geq g(\hat{p}_s),$$

which shows that (TS.18) has no solution if $\bar{\theta}_s = 0$.

We first construct a pair of $(\hat{p}_s, \bar{\theta}_s)$ that satisfies (TS.18) and $s_s(d_s(\bar{\theta}_s) \mid \bar{\theta}_s) = 0$ simultaneously. Let $\hat{p}_s = p_s/Q_s + w_u Q_s/2$ and we shall show that there exists a unique $\bar{\theta}_s > 0$ such that the market clearing condition (TS.18) holds. To see this, rewrite (TS.18)

$$Q_s \bar{F}(\bar{\theta}_s) = g(\hat{p}_s) = g(p_s/Q_s + w_u Q_s/2).$$

Since $g(p_s/Q_s + w_u Q_s/2) < Q_s$ and $\bar{F}(\theta)$ strictly decreases in θ, $Q_s \bar{F}(\bar{\theta}_s) = g(p_s/Q_s + w_u Q_s/2)$ holds for a unique $\bar{\theta}_s > 0$.

We next show $\bar{\theta}_s < \hat{p}_s = p_s/Q_s + w_u Q_s/2$. Since $Q_s \bar{F}(\bar{\theta}_s) = g(\hat{p}_s)$ and $Q_s < \overline{Q}_s = g(p_s/Q_s + w_u Q_s/2)/\bar{F}(p_s/Q_s - w_u Q_s/2)$,

$$\begin{aligned} g(\hat{p}_s) = Q_s \bar{F}(\bar{\theta}_s) < \overline{Q}_s \bar{F}(\bar{\theta}_s) &= g(p_s/Q_s + w_u Q_s/2) \bar{F}(\bar{\theta}_s)/\bar{F}(p_s/Q_s - w_u Q_s/2) \\ &= g(\hat{p}_s) \bar{F}(\bar{\theta}_s)/\bar{F}(p_s/Q_s - w_u Q_s/2), \end{aligned} \tag{TS.24}$$

where the last equality is due to $\hat{p}_s = p_s/Q_s + w_u Q_s/2$. (TS.24) implies that $\bar{F}(\bar{\theta}_s)/\bar{F}(p_s/Q_s + w_u Q_s/2) = \bar{F}(\bar{\theta}_s)/\bar{F}(\hat{p}_s) > 1$ and thus $\bar{\theta}_s < \hat{p}_s = p_s/Q_s + w_u Q_s/2$ due to the strict monotonicity of $\bar{F}(\cdot)$. Since $\bar{\theta}_s < \hat{p}_s - w_u Q_s$, $d_s(\bar{\theta}_s) = 0$ By Lemma TS6. Hence, $s_s(d_s(\bar{\theta}_s) \mid \bar{\theta}_s) = -\frac{1}{2} w_u Q_s^2 - p_s + \hat{p}_s Q_s = 0$ by (TS.16).

So far, we have a pair of $(\hat{p}_s, \bar{\theta}_s)$ that arises as a sharing equilibrium with speculators. We next show that this is the only sharing equilibrium with speculators. First, we show that there does not exist other equilibrium with $\bar{\theta}_s > 0$. Assume there is another sharing equilibrium with speculators with $\bar{\theta}_s = \bar{\theta}_s > 0$. From the proof of necessity, we know that for $\bar{\theta}_s > 0$ the corresponding $\hat{p}_s$ must equal to $p_s/Q_s + w_u Q_s/2$. However, when we set $\hat{p}_s = p_s/Q_s + w_u Q_s/2$ in the constructive proof above, it is shown that (TS.24) holds at a unique $\bar{\theta}_s > 0$. Therefore, we conclude $(\hat{p}_s, \bar{\theta}_s) = (\hat{p}_s, \bar{\theta}_s)$. Second, we show that there exists no equilibrium with $\bar{\theta}_s = 0$. Suppose $\bar{\theta}_s = 0$. This means type-$\bar{\theta}_s$ customers do not value the service at all. They, thus, consume nothing even though they subscribe to it. Hence, $d_s(\bar{\theta}_s) = 0$ and $s_s(d_s(\bar{\theta}_s) \mid \bar{\theta}_s) = -\frac{1}{2} w_u Q_s^2 - p_s + \hat{p}_s Q_s \geq 0$, which implies $\hat{p}_s \geq p_s/Q_s + w_u Q_s/2$. Then, we have

$$Q_s \bar{F}(\bar{\theta}_s) = Q_s > g(p_s/Q_s + w_u Q_s/2) \geq g(\hat{p}_s),$$

which shows that (TS.18) has no solution if $\bar{\theta}_s = 0$. Thus, there is no speculating equilibrium with $\bar{\theta}_s = 0$.

(ii) *Necessity*. Suppose that the sharing market has an unique equilibrium without speculators. Let $\hat{p}_s \geq 0$ be the market clearing price. First, we show $\left(\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s} \frac{\theta-\hat{p}_s+w_uQ_s}{1+w_u} f(\theta)\mathrm{d}\theta + \int_{\hat{p}_s+Q_s}^{\Theta} \frac{\theta-\hat{p}_s+w_oQ_s}{1+w_o} f(\theta)\mathrm{d}\theta\right) \Big/ \bar{F}(\bar{\theta}_s)$ is increasing in $\bar{\theta}_s$. Consider the first derivative in $\bar{\theta}_s$

$$
\begin{aligned}
&\left(\frac{\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s} \frac{\theta-\hat{p}_s+w_uQ_s}{1+w_u} f(\theta)\mathrm{d}\theta + \int_{\hat{p}_s+Q_s}^{\Theta} \frac{\theta-\hat{p}_s+w_oQ_s}{1+w_o} f(\theta)\mathrm{d}\theta}{\bar{F}(\bar{\theta}_s)}\right) \\
=& \frac{-\frac{\bar{\theta}_s-\hat{p}_s+w_uQ_s}{1+w_u}\bar{F}(\bar{\theta}_s) + \int_{\bar{\theta}_s}^{\hat{p}_s+Q_s} \frac{\theta-\hat{p}_s+w_uQ_s}{1+w_u} f(\theta)\mathrm{d}\theta + \int_{\hat{p}_s+Q_s}^{\Theta} \frac{\theta-\hat{p}_s+w_oQ_s}{1+w_o} f(\theta)\mathrm{d}\theta f(\theta)\mathrm{d}\theta}{\bar{F}^2(\bar{\theta}_s)} f(\bar{\theta}_s) \\
\geq& \frac{-\frac{\bar{\theta}_s-\hat{p}_s+w_uQ_s}{1+w_u}\bar{F}(\bar{\theta}_s) + \int_{\bar{\theta}_s}^{\hat{p}_s+Q_s} \frac{\bar{\theta}_s-\hat{p}_s+w_uQ_s}{1+w_u} f(\theta)\mathrm{d}\theta + \int_{\hat{p}_s+Q_s}^{\Theta} Q_s f(\theta)\mathrm{d}\theta f(\theta)\mathrm{d}\theta}{\bar{F}^2(\bar{\theta}_s)} f(\bar{\theta}_s) \\
=& \frac{\int_{\hat{p}_s+Q_s}^{\Theta} Q_s - \frac{\bar{\theta}_s-\hat{p}_s+w_uQ_s}{1+w_u} f(\theta)\mathrm{d}\theta f(\theta)\mathrm{d}\theta}{\bar{F}^2(\bar{\theta}_s)} f(\bar{\theta}_s) > 0,
\end{aligned}
$$

where the first inequality is due to $\theta \geq \bar{\theta}_s$ for $\theta \quad [\bar{\theta}_s, \hat{p}_s + Q_s]$ and $\theta \geq \hat{p}_s + Q_s$ for $\theta \quad [\hat{p}_s + Q_s, \Theta]$, the second inequality is due to $\bar{\theta}_s \leq \hat{p}_s + Q_s$.

Second, we show $\bar{\theta}_s \geq p_s/Q_s - w_uQ_s/2$. Let (TS.17) equals to zero, we have $\hat{p}_s = \bar{\theta}_s + \sqrt{(1+w_u)Q_s^2 - 2(1+w_u)\bar{\theta}_sQ_s + 2(1+w_u)p_s} - Q_s$. Recall that $\bar{\theta}_s \geq \hat{p}_s - w_uQ_s$ by Lemma TS7(ii). Therefore,

$$
\bar{\theta}_s \geq \hat{p}_s - w_uQ_s \qquad \sqrt{(1+w_u)Q_s^2 - 2(1+w_u)\bar{\theta}_sQ_s + 2(1+w_u)p_s} \leq (1+w_u)Q_s \qquad \bar{\theta}_s \geq p_s/Q_s - w_uQ_s/2.
$$

Third, we show $Q_s \geq \overline{Q}_s$. By (TS.19), $Q_s = \left(\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s} \frac{\theta-\hat{p}_s+w_uQ_s}{1+w_u} f(\theta)\mathrm{d}\theta + \int_{\hat{p}_s+Q_s}^{\Theta} \frac{\theta-\hat{p}_s+w_oQ_s}{1+w_o} f(\theta)\mathrm{d}\theta\right) \Big/ \bar{F}(\bar{\theta}_s)$, then it is equivalent to show

$$
\begin{aligned}
&\frac{\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s} \frac{\theta-\hat{p}_s+w_uQ_s}{1+w_u} f(\theta)\mathrm{d}\theta + \int_{\hat{p}_s+Q_s}^{\Theta} \frac{\theta-\hat{p}_s+w_oQ_s}{1+w_o} f(\theta)\mathrm{d}\theta}{\bar{F}(\bar{\theta}_s)} \\
\geq& \frac{\int_{p_s/Q_s-w_uQ_s/2}^{\hat{p}_s+Q_s} \frac{\theta-\hat{p}_s+w_uQ_s}{1+w_u} f(\theta)\mathrm{d}\theta + \int_{\hat{p}_s+Q_s}^{\Theta} \frac{\theta-\hat{p}_s+w_oQ_s}{1+w_o} f(\theta)\mathrm{d}\theta}{\bar{F}(p_s/Q_s - w_uQ_s/2)} \\
\geq& \frac{\int_{p_s/Q_s-w_uQ_s/2}^{p_s/Q_s+w_uQ_s/2+Q_s} \frac{\theta-p_s/Q_s+w_uQ_s/2}{1+w_u} f(\theta)\mathrm{d}\theta + \int_{p_s/Q_s+w_uQ_s/2+Q_s}^{\Theta} \frac{\theta-p_s/Q_s-w_uQ_s/2+w_oQ_s}{1+w_o} f(\theta)\mathrm{d}\theta}{\bar{F}(p_s/Q_s - w_uQ_s/2)} = \overline{Q}_s,
\end{aligned}
$$

where the first inequality is due to $\left(\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s} \frac{\theta-\hat{p}_s+w_uQ_s}{1+w_u} f(\theta)\mathrm{d}\theta + \int_{\hat{p}_s+Q_s}^{\Theta} \frac{\theta-\hat{p}_s+w_oQ_s}{1+w_o} f(\theta)\mathrm{d}\theta\right) \Big/ \bar{F}(\bar{\theta}_s)$ is increasing in $\bar{\theta}_s$ and $\bar{\theta}_s \geq p_s/Q_s - w_uQ_s/2$, the second inequality is due to $\hat{p}_s \leq p_s/Q_s + w_uQ_s/2$ and $\int_{p_s/Q_s-w_uQ_s/2}^{\hat{p}_s+Q_s} \frac{\theta-\hat{p}_s+w_uQ_s}{1+w_u} f(\theta)\mathrm{d}\theta + \int_{\hat{p}_s+Q_s}^{\Theta} \frac{\theta-\hat{p}_s+w_oQ_s}{1+w_o} f(\theta)\mathrm{d}\theta$ is decreasing in $\hat{p}_s$.

Last, we prove $p_s/Q_s + Q_s/2 \leq \Theta$ by contradiction. Suppose $p_s/Q_s + Q_s/2 > \Theta$. Let $l(\theta) = \theta + \sqrt{(1+w_u)Q_s^2 - 2(1+w_u)Q_s\theta + 2(1+w_u)p_s}$, then

$$
\begin{aligned}
l(\Theta) &= \Theta + \sqrt{(1+w_u)Q_s^2 - 2(1+w_u)Q_s\Theta + 2(1+w_u)p_s} \\
&> \Theta + \sqrt{(1+w_u)Q_s^2 - 2(1+w_u)Q_s\left(p_s/Q_s + Q_s/2\right) + 2(1+w_u)p_s} = \Theta,
\end{aligned}
$$

which implies $\hat{p}_s = l(\bar{\theta}_s) - Q_s > \Theta - Q_s$. We thus reach a contradiction since $\hat{p}_s + Q_s \leq \Theta$.

Sufficiency. Suppose $Q_s \geq \overline{Q}_s$ and $p_s/Q_s + Q_s/2 \leq \Theta$. To ensure the existence of a sharing market without speculators with a unique market clearing price, we need to show that (TS.19) and

$\frac{(\theta-\hat{p}_s+w_uQ_s)^2}{2(1+w_u)}-\frac{1}{2}w_uQ_s^2-p_s+\hat{p}_sQ_s=0$ simultaneously hold with a unique set of $\hat{p}_s$ and $\bar{\theta}_s\geq\hat{p}_s-w_uQ_s$. Let

$$\hat{p}_s=l(\bar{\theta}_s)-Q_s=\bar{\theta}_s+\sqrt{(1+w_u)Q_s^2-2(1+w_u)\bar{\theta}_sQ_s+2(1+w_u)p_s}-Q_s, \tag{TS.25}$$

which implies $\frac{(\theta-\hat{p}_s+w_uQ_s)^2}{2(1+w_u)}-\frac{1}{2}w_uQ_s^2-p_s+\hat{p}_sQ_s=0$. We next show there exists a unique solution $p_s/Q_s-w_uQ_s/2\leq\bar{\theta}_s<p_s/Q_s+Q_s/2\leq\Theta$ such that

$$Q_s=\left(\int_{\bar{\theta}_s}^{\hat{p}_s+Q_s}\frac{\theta-\hat{p}_s+w_uQ_s}{1+w_u}f(\theta)\mathrm{d}\theta+\int_{\hat{p}_s+Q_s}^{\Theta}\frac{\theta-\hat{p}_s+w_oQ_s}{1+w_o}f(\theta)\mathrm{d}\theta\right)\Big/\bar{F}(\bar{\theta}_s),$$

which is equivalent to (TS.19). We can verify this by three steps in the followings. First, we show $L(\theta,l(\theta)-Q_s)$ is strictly increasing in $\theta\geq p_s/Q_s-w_uQ_s/2$, where $L(\theta,\hat{p}_s)=(\int_{\theta}^{\hat{p}_s+Q_s}\frac{\theta-\hat{p}_s+w_uQ_s}{1+w_u}f(\theta)\mathrm{d}\theta+\int_{\hat{p}_s+Q_s}^{\Theta}\frac{\theta-\hat{p}_s+w_oQ_s}{1+w_o}f(\theta)\mathrm{d}\theta)/\bar{F}(\theta)$. Recall $L(\theta,\hat{p}_s)$ is strictly increasing in θ and decreasing in $\hat{p}_s$, then it is sufficiently to prove $l(\theta)$ is strictly decreasing in $\theta\geq p_s/Q_s-w_uQ_s/2$. That is

$$\begin{aligned}l\ (\theta)&=1-\frac{(1+w_u)Q_s}{\sqrt{(1+w_u)Q_s^2-2(1+w_u)\bar{\theta}_sQ_s+2(1+w_u)p_s}}\\&\leq1-\frac{(1+w_u)Q_s}{\sqrt{(1+w_u)Q_s^2-2(1+w_u)\left(p_s/Q_s-w_uQ_s/2\right)Q_s+2(1+w_u)p_s}}=0.\end{aligned}$$

Second, we show $Q_s\geq L(p_s/Q_s-w_uQ_s/2,l(p_s/Q_s-w_uQ_s/2)-Q_s)$. This comes from $L(p_s/Q_s-w_uQ_s/2,l(p_s/Q_s-w_uQ_s/2)-Q_s)=\overline{Q}_s$ and $Q_s\geq\overline{Q}_s$ immediately. Third, we show $Q_s<L(p_s/Q_s+Q_s/2,l(p_s/Q_s+Q_s/2)-Q_s)$. Note $l(p_s/Q_s+Q_s/2)=p_s/Q_s+Q_s/2$, then $L(p_s/Q_s+Q_s/2,l(p_s/Q_s+Q_s/2)-Q_s)=\int_{p_s/Q_s+Q_s/2}^{\Theta}\frac{\theta-p_s/Q_s+Q_s/2+w_oQ_s}{1+w_o}f(\theta)\mathrm{d}\theta/\bar{F}(p_s/Q_s+Q_s/2)>Q_s$ due to the Mean Value Theorem.

At last, we show $\bar{\theta}_s\geq\hat{p}_s-w_uQ_s$. By (TS.25),

$$\begin{aligned}\hat{p}_s=&\bar{\theta}_s+\sqrt{(1+w_u)Q_s^2-2(1+w_u)\bar{\theta}_sQ_s+2(1+w_u)p_s}-Q_s\\\leq&\bar{\theta}_s+\sqrt{(1+w_u)Q_s^2-2(1+w_u)(p_s/Q_s-w_uQ_s/2))Q_s+2(1+w_u)p_s}-Q_s=\bar{\theta}_s-w_uQ_s,\end{aligned}$$

where the inequality is due to $\bar{\theta}_s\geq p_s/Q_s-w_uQ_s/2$.

Lemma TS8. *Assume that the service provider offers a nonlinear contract $(p_n,Q_n,\hat{p}_n)$. If a type-θ customer subscribes, her optimal consumption level satisfies*

$$d_n(\theta)=\begin{cases}\frac{\theta+w_uQ_n}{1+w_u}, & \text{if } 0\leq\theta<Q_n\\ Q_n, & \text{if } Q_n\leq\theta<\hat{p}_n+Q_n\\ \frac{\theta+w_oQ_n-\hat{p}_n}{1+w_o}, & \text{otherwise.}\end{cases} \tag{TS.26}$$

Proof of Lemma TS8. We can write $s_n(d_n\,|\,\theta)$ in (25) as

$$s_n(d_n\,|\,\theta)=\begin{cases}-\frac{1}{2}(1+w_u)d_n^2+(\theta+w_uQ_n)d_n-\frac{1}{2}w_uQ_n^2-p_n, & \text{if } d_n<Q_n\\ -\frac{1}{2}(1+w_o)d_n^2+(\theta+w_oQ_n-\hat{p}_n)d_n-\frac{1}{2}w_oQ_n^2+\hat{p}_nQ_n-p_n, & \text{if } d_n\geq Q_n.\end{cases}$$

Note that $s_n(d_n\,|\,\theta)$ is concave in d_n when $d_n<Q_n$ and $d_n\geq Q_n$, respectively. Then, conditional on the fact that a customer has already subscribed to the service, i.e., $s_n(d_n\,|\,\theta)\geq0$, we can derive her demand by the first order condition (FOC). Hence,

$$\frac{\partial s_n}{\partial d_n}=\begin{cases}\theta+w_uQ_n-(1+w_u)d_n=0, & \text{if } d_n<Q_n & \text{(TS.27)}\\ \theta+w_oQ_n-\hat{p}_n-(1+w_o)d_n=0, & \text{if } d_n\geq Q_n & \text{(TS.28)}\end{cases}$$

which implies

$$d_n = \begin{cases} \dfrac{\theta + w_u Q_n}{1 + w_u}, & \text{if } d_n < Q_n \qquad \text{(TS.29)} \\ \dfrac{\theta + w_o Q_n - \hat{p}_n}{1 + w_o}, & \text{if } d_n \geq Q_n. \qquad \text{(TS.30)} \end{cases}$$

We next write d_n as function of customer type θ. First, if $\theta < Q_n$, it is easy to see that $d_n(\theta) = \frac{\theta + w_u Q_n}{1+w_u} < Q_n$ by (TS.29). Second, if $\theta \geq \hat{p}_n + Q_n$, then by (TS.30) we have $d_n(\theta) = \frac{\theta + w_o Q_n - \hat{p}_n}{1+w_o} \geq Q_n$. At last, we show that $d_n(\theta) = Q_n$ if $Q_n \leq \theta < \hat{p}_n + Q_n$:

(i) Assume a type-θ customer consumes $d_n < Q_n$. By (TS.27), $\frac{\partial s_n}{\partial d_n} = \theta + w_u Q_n - (1+w_u) d_n \geq Q_n + w_u Q_n - (1+w_u) d_n = (1+w_u)(Q_n - d_n) > 0$, then the customer prefers increasing her demand to Q_n, i.e., $d_n(\theta) = Q_n$.

(ii) Assume a type-θ customer consumes $d_n \geq Q_n$. By (TS.28), $\frac{\partial s_n}{\partial d_n} = \theta + w_o Q_n - \hat{p}_n - (1+w_o) d_n < \hat{p}_n + Q_n + w_o Q_n - \hat{p}_n - (1+w_o) d_n = (1+w_o)(Q_n - d_n) \leq 0$, then the customer prefers decreasing her demand to Q_n, i.e., $d_n(\theta) = Q_n$.

Lemma TS9. *Assume that the service provider offers a nonlinear contract $(p_n, Q_n, \hat{p}_n)$. A nonzero fraction of customers subscribe if and only if $p_n \leq \frac{1}{2}(\Theta - \hat{p}_n)^2 + \hat{p}_n Q_n$. Specifically, there exists a unique $\bar{\theta}_n$ such that customers subscribe to the service if and only if $\theta \geq \bar{\theta}_n$, where*

$$\bar{\theta}_n = \begin{cases} \sqrt{(1+w_u)(2p_n + w_u Q_n^2)} - w_u Q_n, & \text{if } 0 \leq p_n < \frac{Q_n^2}{2} \\ p_n / Q_n + Q_n / 2, & \text{if } \frac{Q_n^2}{2} \leq p_n < \hat{p}_n Q_n + \frac{Q_n^2}{2} \\ \hat{p}_n + \sqrt{(1+w_o)(2(p_n - \hat{p}_n Q_n) + w_o Q_n^2)} - w_o Q_n, & \text{if } \hat{p}_n Q_n + \frac{Q_n^2}{2} \leq p_n \leq \frac{(\Theta - \hat{p}_n + w_o Q_n)^2}{2(1+w_o)} + \hat{p}_n Q_n - \frac{w_o Q_n^2}{2}. \end{cases} \tag{TS.31}$$

Proof of Lemma TS9. By Lemma TS8, we can write $s_n(d_n \mid \theta)$ in (25) as

$$s_n(d_n(\theta) \mid \theta) = \begin{cases} \dfrac{(\theta + w_u Q_n)^2}{2(1+w_u)} - \dfrac{1}{2} w_u Q_n^2 - p_n, & \text{if } 0 \leq \theta < Q_n \qquad \text{(TS.32)} \\ \theta Q_n - \dfrac{1}{2} Q_n^2 - p_n, & \text{if } Q_n \leq \theta < \hat{p}_n + Q_n \qquad \text{(TS.33)} \\ \dfrac{(\theta + w_o Q_n - \hat{p}_n)^2}{2(1+w_o)} - \dfrac{1}{2} w_o Q_n^2 - (p_n - \hat{p}_n Q_n), & \text{if } \hat{p}_n + Q_n \leq \theta \leq \Theta. \qquad \text{(TS.34)} \end{cases}$$

If $p_n > \frac{(\Theta - \hat{p}_n + w_o Q_n)^2}{2(1+w_o)} + \hat{p}_n Q_n - \frac{w_o Q_n^2}{2}$, $s_n(d_n(\theta = \Theta) \mid \theta = \Theta) < 0$. Since $s_n(d_n(\theta) \mid \theta)$ strictly increases in θ, no customers earn positive surplus and hence there are no subscribers. On the contrary, if $0 \leq p_n \leq \frac{(\Theta - \hat{p}_n + w_o Q_n)^2}{2(1+w_o)} + \hat{p}_n Q_n - \frac{w_o Q_n^2}{2}$, $s_n(d_n(\theta = 0) \mid \theta = 0) \leq 0$ and $s_n(d_n(\theta = \Theta) \mid \theta = \Theta) \geq 0$. Therefore, there exists a unique $\bar{\theta}_n$ such that $s_n(d_n(\theta) \mid \theta) = 0$ and customers subscribe to the service if and only if $\theta \geq \bar{\theta}_n$.

Next, we characterize $\bar{\theta}_n$. First, let us consider the case $0 \leq p_n < \frac{1}{2} Q_n^2$. For any customer of type θ $[Q_n, \hat{p}_n + Q_n)$, $s_n(d_n(\theta) \mid \theta) = \theta Q_n - \frac{1}{2} Q_n^2 - p_n > \theta Q_n - Q_n^2 \geq 0$ by eq. (TS.33). For any customer of type θ $[\hat{p}_n + Q_n, \Theta]$, $s_n(d_n(\theta) \mid \theta) = \frac{(\theta + w_o Q_n - \hat{p}_n)^2}{2(1+w_o)} - \frac{1}{2} w_o Q_n^2 - (p_n - \hat{p}_n Q_n) \geq Q_n^2/2 + \hat{p}_n Q_n - p_n > 0$ by eq. (TS.34). In other words, all customers of types θ $[Q_n, \Theta]$ choose to subscribe. Thus, the continuity of s_n implies $\bar{\theta}_n < Q_n$: setting $s_n(d_n(\theta) \mid \theta)$ in eq. (TS.32) to be zero, we have $\bar{\theta}_n = \sqrt{(1+w_u)(2p_n + w_u Q_n^2)} - w_u Q_n$.

Second, consider $\frac{1}{2}Q_n^2 \leq p_n < \hat{p}_n Q_n + \frac{1}{2}Q_n^2$. For any customer of type $\theta \quad [0, Q_n)$, $s_n(d_n(\theta) \mid \theta) = \frac{(\theta + w_u Q_n)^2}{2(1+w_u)} - \frac{1}{2}w_u Q_n^2 - p_n < \frac{1}{2}Q_n^2 - p_n \leq 0$ by eq. (TS.32). In other words, no customers of types $\theta \quad [0, Q_n)$ choose to subscribe. For any customer of type $\theta \quad [\hat{p}_n + Q_n, \Theta]$, $s_n(d_n(\theta) \mid \theta) = \frac{(\theta + w_o Q_n - \hat{p}_n)^2}{2(1+w_o)} - \frac{1}{2}w_o Q_n^2 - (p_n - \hat{p}_n Q_n) \geq \frac{1}{2}Q_n^2 - (p_n - \hat{p}_n Q_n) > 0$ by eq. (TS.34). In other words, all customers of types $\theta \quad [\hat{p}_n + Q_n, \Theta]$ choose to subscribe. Thus, the continuity of s_n implies $Q_n \leq \bar{\theta}_n < \hat{p}_n + Q_n$: setting $s_n(d_n(\theta) \mid \theta)$ in eq. (TS.33) to be zero, we have $\bar{\theta}_n = p_n/Q_n + Q_n^2/2$.

At last, consider $\hat{p}_n Q_n + \frac{1}{2}Q_n^2 \leq p_n \leq \frac{(\Theta - \hat{p}_n + w_o Q_n)^2}{2(1+w_o)} + \hat{p}_n Q_n - \frac{w_o Q_n^2}{2}$. For any customer of type $\theta \quad [0, Q_n)$, $s_n(d_n(\theta) \mid \theta) = \frac{(\theta + w_u Q_n)^2}{2(1+w_u)} - \frac{1}{2}w_u Q_n^2 - p_n < \frac{1}{2}Q_n^2 - p_n \leq 0$ by eq. (TS.32). In other words, no customers of types $\theta \quad [0, Q_n)$ choose to subscribe. For any customer of type $\theta \quad [Q_n, \hat{p}_n + Q_n)$, $s_n(d_n(\theta) \mid \theta) = \theta Q_n - \frac{1}{2}Q_n^2 - p_n < (\hat{p}_n + Q_n)Q_n - \frac{1}{2}Q_n^2 - p_n = \hat{p}_n Q_n + \frac{1}{2}Q_n^2 - p_n \leq 0$ by eq. (TS.33). In other words, no customers of types $\theta \quad [Q_n, \hat{p}_n + Q_n)$ choose to subscribe. Thus, the continuity of s_n implies $\hat{p}_n + Q_n \leq \bar{\theta}_n \leq \Theta$: setting $s_n(d_n(\theta) \mid \theta)$ in eq. (TS.34) to be zero, we have $\bar{\theta}_n = \hat{p}_n + \sqrt{(1+w_o)(2(p_n - \hat{p}_n Q_n) + w_o Q_n^2)} - w_o Q_n$.

LEMMA TS10. *(i) For any uniform distribution on $[0, a]$, denote Π_s^a and Π_n^a as the optimal revenue under sharing and nonlinear contract, respectively. Then $\Pi_s^a = a^2 \Pi_s^1$ and $\Pi_n^a = a^2 \Pi_n^1$.*

(ii) For any exponential distribution with probability density function $f(\theta) = \lambda e^{-\lambda\theta}$, denote Π_s^λ and Π_n^λ as the optimal revenue under sharing and nonlinear contract, respectively. Then $\Pi_s^\lambda = \Pi_s^1/\lambda^2$ and $\Pi_n^\lambda = \Pi_n^1/\lambda^2$.

Proof of Lemma TS10. (i) First, we consider the sharing contract. Denote the optimal contract (p_s^a, Q_s^a) with the resulting equilibrium market clearing price $\hat{p}_s^a$, where p_s^a, Q_s^a, $\hat{p}_s^a$ are given by (24). Note $f(\theta) = 1/a$ and $\bar{F}(\theta) = (a - \theta)/a$, then (24) is equivalent to

$$\begin{cases} \left(\frac{\hat{p}_s/a}{1+w_u} + \frac{1 - \hat{p}_s/a - Q_s/a}{1+w_o}\right)\left((1 - Q_s/a)(\hat{p}_s/a - (1+w_u)Q_s/a) + \frac{(\hat{p}_s/a)^2 + (1+w_u)(Q_s/a)^2}{2}\right) = \frac{(\hat{p}_s/a)^2(1 - Q_s/a)}{1+w_u}, \\ (Q_s/a)(1 - Q_s/a) = \frac{(w_u - w_o)(Q_s/a)^2}{2} - \frac{(Q_s/a - \hat{p}_s/a + w_u Q_s/a)^2}{2(1+w_u)} + \frac{(1 - \hat{p}_s/a + w_o Q_s/a)^2}{2(1+w_o)}, \\ \hat{p}_s/a = \sqrt{(1+w_u)(2p_s/a^2 - (Q_s/a)^2)}. \end{cases}$$

Thus p_s^a/a^2, Q_s^a/a, and $\hat{p}_s^a/a$ are independent on a. Let $x_s = p_s^a/a^2$ and $y_s = Q_s^a/a$. Recall the threshold of subscribing under optimal nonlinear contract $\bar{\theta}_s^a$ equals to Q_s^a by Proposition 8(ii). Hence, the optimal revenue is $\Pi_s^a = p_s^a \bar{F}(Q_s^a) = a^2 x_s(1 - y_s) = a^2 \Pi_s^1$.

Second, we consider the nonlinear contract. Denote $\bar{\theta}_n^a$ as the threshold of subscribing under optimal nonlinear contract, which is given by

$$\frac{(1+w_o)^2\bar{\theta}_n^2 - \left(\frac{\int_{\bar{\theta}_n}^{\Theta} \theta f(\theta) \mathrm{d}\theta}{\bar{F}(\bar{\theta}_n)} - \bar{\theta}_n\right)^2}{2\left[(1+w_o)^2\bar{\theta}_n - \frac{\int_{\bar{\theta}_n}^{\Theta} \theta f(\theta) \mathrm{d}\theta}{\bar{F}(\bar{\theta}_n)} + \bar{\theta}_n\right]} = \frac{\bar{F}(\bar{\theta}_n)}{f(\bar{\theta}_n)}$$

due to Proposition 9(i). Note $f(\theta) = 1/a$ and $\bar{F}(\theta) = (a - \theta)/a$, then the equation above is equivalent to $[12(1+w_o)^2 + 3](\bar{\theta}_n/a)^2 - [8(1+w_o)^2 + 6](\bar{\theta}_n/a) + 3 = 0$. Thus $\bar{\theta}_n^a/a$ is independent on a. Let $x_n = \bar{\theta}_n^a/a$, then the optimal overage rate $\hat{p}_n^a = \frac{a(1 - x_n)}{2(1+w_o)}$ by Proposition 9(ii). From the proof of Theorem 4(i), the optimal revenue under nonlinear contract is $\Pi_n^a = (\frac{1}{2}\bar{\theta}_n^{a2} + \frac{1}{2}\hat{p}_n^{a2})\bar{F}(\bar{\theta}_n^a) = \frac{a^2}{2}(x_n^2 + \frac{(1 - x_n)^2}{4(1+w_o)^2})(1 - x_n) = a^2 \Pi_n^1$.

(ii) First, we consider the sharing contract. Denote the optimal contract $(p_s^\lambda, Q_s^\lambda)$ with the resulting equilibrium market clearing price $\hat{p}_s^\lambda$, where p_s^λ, Q_s^λ, $\hat{p}_s^\lambda$ are given by (24). Note $f(\theta) = \lambda e^{-\lambda\theta}$ and $\bar{F}(\theta) = e^{-\lambda\theta}$, then (24) is equivalent to

$$\begin{cases} \left(\frac{1-e^{-\lambda\hat{p}_s}}{1+w_u} + \frac{e^{-\lambda\hat{p}_s}}{1+w_o}\right)\left(\lambda\hat{p}_s - (1+w_u)\lambda Q_s + \frac{(\lambda\hat{p}_s)^2+(1+w_u)(\lambda Q_s)^2}{2}\right) = \frac{(\lambda\hat{p}_s)^2}{1+w_u}, \\ \lambda Q_s = \frac{(1+w_u)\lambda Q_s - \lambda\hat{p}_s + 1 - e^{-\lambda\hat{p}_s}}{1+w_u} + \frac{e^{-\lambda\hat{p}_s}}{1+w_o}, \\ \lambda\hat{p}_s = \sqrt{(1+w_u)(2\lambda^2 p_s - (\lambda Q_s)^2)}. \end{cases}$$

Thus $\lambda^2 p_s^\lambda$, λQ_s^λ, and $\lambda\hat{p}_s^\lambda$ are independent on λ. Let $x_s = \lambda^2 p_s^\lambda$ and $y_s = \lambda Q_s^\lambda$. Recall the threshold of subscribing under optimal nonlinear contract $\bar{\theta}_s^\lambda$ equals to Q_s^λ by Proposition 8(ii). Hence, the optimal revenue is $\Pi_s^\lambda = p_s^\lambda \bar{F}(Q_s^\lambda) = x_s e^{-y_s}/\lambda^2 = \Pi_s^1/\lambda^2$.

Second, we consider the nonlinear contract. Denote $\bar{\theta}_n^\lambda$ as the threshold of subscribing under optimal nonlinear contract, which is given by

$$\frac{(1+w_o)^2\bar{\theta}_n^2 - \left(\frac{\int_{\bar{\theta}_n}^{\Theta}\theta f(\theta)\mathrm{d}\theta}{\bar{F}(\bar{\theta}_n)} - \bar{\theta}_n\right)^2}{2\left[(1+w_o)^2\bar{\theta}_n - \frac{\int_{\bar{\theta}_n}^{\Theta}\theta f(\theta)\mathrm{d}\theta}{\bar{F}(\bar{\theta}_n)} + \bar{\theta}_n\right]} = \frac{\bar{F}(\bar{\theta}_n)}{f(\bar{\theta}_n)}$$

due to Proposition 9(i). Note $f(\theta) = \lambda e^{-\lambda\theta}$ and $\bar{F}(\theta) = e^{-\lambda\theta}$, then the equation above is equivalent to $(1+w_o)^2(\lambda\bar{\theta}_n^\lambda)^2 - 2(1+w_o)^2\lambda\bar{\theta}_n^\lambda + 1$. Thus $\lambda\bar{\theta}_n^\lambda$ is independent on λ. Let $x_n = \lambda\bar{\theta}_n^\lambda$, then the optimal overage rate $\hat{p}_n^\lambda = \frac{1}{\lambda(1+w_o)}$ by Proposition 9(ii). From the proof of Theorem 4(i), the optimal revenue under nonlinear contract is $\Pi_n^\lambda = (\frac{1}{2}\bar{\theta}_n^{\lambda 2} + \frac{1}{2}\hat{p}_n^{\lambda 2})\bar{F}(\bar{\theta}_n^\lambda) = \frac{1}{2\lambda^2}(x_n^2 + \frac{1}{(1+w_o)^2})e^{-x_n} = \Pi_n^1/\lambda^2$.