Information Systems Research

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

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INFORMATION SYSTEMS RESEARCH
Vol. 36, No. 1, March 2025, pp. 21–40
ISSN 1047-7047 (print), ISSN 1526-5536 (online)
https://pubsonline.informs.org/journal/isre

Mispricing and Algorithm Trading

Lihong Zhang,[a] **Xiaoquan (Michael) Zhang**[a,b,*]

[a] School of Economics and Management, Tsinghua University, Beijing 100084, China; [b] CUHK Business School, Chinese University of Hong Kong, Shatin, NT, Hong Kong Special Administrative Region, China

*Corresponding author

Contact: zhanglh2@sem.tsinghua.edu.cn, https://orcid.org/0009-0003-0439-8124 (LZ); zhang@cuhk.edu.hk, https://orcid.org/0000-0003-0690-2331 (X(M)Z)

Received: November 9, 2021
Revised: July 30, 2022; May 20, 2023
Accepted: July 9, 2023
Published Online in Articles in Advance: February 19, 2024

https://doi.org/10.1287/isre.2021.0570

Abstract. The widespread adoption of information technology has fundamentally transformed the way information is processed in the financial market. One such technological advancement is algorithm trading, which allows traders to develop sophisticated strategies based on historical price data. This raises important questions: Do these algorithm trading strategies contribute to market instability? When do they yield profits for different market participants? To address these questions, we must move beyond the efficient market hypothesis, as this theory would suggest that such strategies yield no profit due to market efficiency. Instead, we explicitly incorporate initial market mispricing into our analysis and develop a stylized continuous-time model of algorithm feedback trading to investigate market outcomes. Our model yields closed-form solutions, enabling us to assess the degree to which the price diverges from the efficient level. We discover that algorithmic trading, when combined with initial market mispricing, can lead to significant market volatility, resulting in financial bubbles and crashes. However, this scenario only occurs when there is overpricing and the algorithm traders collectively employ a strategy that enlarges the mispricing. Depending on the initial mispricing in the form of underpricing or overpricing, different algorithm trading strategies (positive or negative) have different market impact, profitability, and policy implications.

History: Ram Gopal, Senior Editor; Yan Huang, Associate Editor.
Funding: L. Zhang received financial support from the National Key Research and Development Program of China [Grant 2007CB814902], the National Natural Science Foundation of China [Grant 70671061], and the Tsinghua University Initiative Scientific Research Program [Grant 2021THZWJC28]. X. (M.). Zhang received financial support from the Hong Kong Research Grant Council [Grants GRF 14500521, 165052947, 14501320, and 14503818; and TRS: T31-604/18-N].
Supplemental Material: The online appendix is available at https://doi.org/10.1287/isre.2021.0570.

Keywords: mispricing • algorithm trading • fintech • market efficiency • financial trading

1. Introduction

Constant innovation and the use of information technologies (IT) have enabled the digital transformation of the financial market and have supported its participants in remaining competitive (Fan et al. 2000, Dewan and Mendelson 2001, Hendershott et al. 2021). One of the most prominent applications of IT in the financial market is algorithm trading (Stoll 2006, Weber 2006, Lucas et al. 2009). Algorithm trading is the use of computers, mathematical models, and high-speed networks to automate the buying and selling of financial assets.

Algorithm trading is one of the earliest areas in which financial technology (FinTech) has an application in the financial market (Hendershott et al. 2021). Early firm-level studies examine the impact of IT on competition and market liquidity (Dewan and Mendelson 2001, Bakos et al. 2005). Following the wave of firm-level adoption of trading platforms, data-driven financial modeling quickly replaced the old way of trading (Clemons and Weber 1996, 1997, Weber 2006, Lucas et al. 2009). Ever since the beginning of the internet, it was clear that, given its implications on information transparency and information disintermediation (Clemons et al. 2002), the internet would play a significant role in changing the financial market (Fan et al. 2000). There is therefore increased need to understand how such technologies are used in reality and how they change the market. Dewan and Mendelson (1998) argue that due to the adoption of quantitative trading tools by various players in the market, there is a significant development in IT infrastructure investments and, as a result, higher-frequency trading. Mendelson and Tunca (2003) show that liquidity traders indeed can benefit from the information available in such FinTech systems. Agarwal et al. (2017) and Shangguan et al. (2022) report that online searches and composite information processing can offer important stock return predictability implications. There is also a growing literature on the impact of IT on the information environment for investors (e.g., Xu and Zhang 2013; Li et al. 2018; Havakhor et al. 2019a, b; Ge et al. 2021).

How does algorithm trading affect the equilibrium of the financial market? When are algorithm trading strategies profitable? We answer these questions in this study.

Each time when there is a financial market crash, there are discussions on how algorithm trading may be the culprit responsible for large market volatilities. The 2020 financial market crash is no exception. During the short one-month period from February 19 to March 23, the S&P 500 index fell by 33.9%. Multiple media channels pointed fingers at algorithm trading. For example, *Fortune* magazine holds the opinion that algorithm trading programs can make bad stock market days even worse.[1] The website MarketWatch.com wrote: "What's driving the speed and severity of the bear market is the escalation of algorithmic trading, which is more prevalent than it was during the Great Recession in 2008."[2] Jarrow and Protter (2012) find that algorithm trading can disrupt financial market efficiency. Unlike arbitrageurs, who eliminate mispricing, algorithm traders can inadvertently create and exploit mispricing to the detriment of regular investors. Mispricing therefore arises from the collective, independent actions of algorithm traders, coordinated through a shared signal.

So far, the answer is not clear. Hendershott et al. (2011) conduct one of the earliest empirical tests and show that algorithm trading can actually improve liquidity and enhance the informativeness of price quotes. In a follow-up study, Hendershott and Riordan (2013) show that algorithm trading consumes liquidity when the bid-ask spreads are narrow and supplies liquidity when the market has lower liquidity. Consistent with these findings, Chaboud et al. (2014) find that algorithm trading can bring an improvement in price efficiency. Kirilenko and Lo (2013) offer an early review of the challenges and opportunities that algorithm trading brings to the financial industry, as well as its regulators. Different from the accusations from the public media, these studies suggest that algorithm trading may contribute to the efficiency of the market. At the same time, some other works challenge this view. Weller (2017) finds that increased algorithm trading is associated with a decreased amount of information in prices. In other words, algorithm trading may reduce price informativeness at the same time when it translates information into prices. Zhang and Zhang (2015) develop an analytical model and show that because algorithmic feedback trading does not bring additional information into the market, it cannot change the market in terms of the price process.

Establishing algorithm trading's impact on financial market's stability is a very challenging task because it is hard to fully control all factors that affect the relationship between algorithm trading and financial asset price changes. This problem is even more pronounced when the market is undergoing severe price changes surrounding a financial crisis. In this study, we examine an analytical model to understand the effect of algorithm trading on market stability. The model is based on a framework frequently used in the literature to study financial market equilibria (Kyle 1985, Hong and Stein 1999, Zhang and Zhang 2015).

Different from prior works in the literature, we explicitly model mispricing in this study and examine how algorithm trading will interact with it. Mispricing is modeled as the divergence between the market price of a financial asset and its efficient market price. This formulation can be considered as a relaxation of the efficient market hypothesis and an extension of the prior works that are based on this hypothesis (Kyle 1985, Hong and Stein 1999, Zhang and Zhang 2015).

If the market is efficient, there cannot exist bubbles and crashes, because in an efficient market, all available information is perfectly incorporated into the price at any point of time.[3] When discussing the 1987 market crash, Malkiel (2003, p. 73) wrote: "…the stock market lost about one-third of its value from early to mid-October 1987 with essentially no change in the general economic environment. How could market prices be efficient both at the start of October and during the middle of the month?" To study whether and how algorithm trading may lead to bubbles and crashes, it is therefore necessary to relax the market efficiency assumption and allow the existence of mispricing. A model of algorithm trading without mispricing will degenerate to the scenario studied in Zhang and Zhang (2015).

In this study, we first explicitly model mispricing and then explore subsequent interplays between rational traders. With this framework, we examine mispricing's impact on market outcomes and specifically offer insights on the profitability of algorithm trading and show how market bubbles and crashes are created.

There are a lot of discussions on how mispricing combined with algorithmic trading can lead to market meltdowns. For example, a *Washington Post* article calls the combination of algorithm trading and mispricing "herd behavior on steroids."[4] The article argues that: "The truth is that the market is just as irrational and divorced from fundamentals on the way up as it is on the way down. It is in the nature of markets more so today than ever, as a result of the computerized high-frequency trading strategies of the Wall Street wise guys." The article attributes the volatile market outcome to algorithm trading and suggests that mispricing is inseparable from algorithm trading. Similarly, a Bloomberg article reports that Adair Turner, the chairman of the United Kingdom's Financial Services Authority, believes "the rise of algorithmic trading may cause markets to be more volatile and securities to be mispriced."[5]

The International Organization of Securities Commissions (IOSCO) Technical Committee released a report in July 2011 about algorithm trading's impact on the financial system.[6] This report viewed mispricing and algorithm trading as the culprits of the Flash Crash of May 2010. In the same year, the Commodity Futures Trading Commission (CFTC) published a paper sharing the same view (Kirilenko et al. 2011).

From a legal perspective, Yadav (2015) argues that algorithmic trading undermines price's role in allocative efficiency through two key mechanisms. Firstly, the systemic model risk in algorithmic markets, stemming from the inherent inability of preset programming to fully capture real-world trading nuances, creates significant costs. This leads to an inefficient focus on short-term trading with limited capital allocation relevance. Secondly, the competitive dynamics of high-speed algorithmic markets discourage participation of informed traders, traditionally crucial for informational contributions. As a result, according to the author, any small mispricing can be significantly enlarged by algorithm trading to move the market further away from the efficient level.

Using simulated algorithmic trading to explore its influence on asset markets, Mukerji et al. (2019) find that statistical arbitrage can lead to and exacerbate mispricing and cause significant divergence of price from fundamental values. Jain et al. (2021) challenge this view and show that an increase in algorithm trading leads to less significant and short-lived price deviations from exchange-traded funds' (ETFs') net asset values. Algorithm traders' arbitrage strategies contribute to reducing these deviations, suggesting an enhancement in market efficiency. Additionally, algorithmic trading augments ETF liquidity. A recent survey examines emerging research on human-algorithmic trading interactions in experimental markets (Bao et al. 2022). The analysis shows that the profitability of algorithmic traders versus human traders hinges significantly on the underlying assets' deviation from the fundamentals and the level of market inefficiency. Corgnet et al. (2023) examine a similar question and find that both limit-order and market-order algorithms contribute to better price efficiency and reduced volatility.

Mispricing can take many different forms and was widely reported in prior research. For example, Rashes (2001) reports that news about MCI Communications (then with a ticker symbol "MCIC") had significant effects on Massmutual Corporate Investors, a fund traded on the New York Stock Exchange (NYSE) with the ticker symbol "MCI." Similarly, Huberman and Regev (2001) document that a newspaper report of a previously known cure for cancer caused the stock price of a drug company to rise. Recently, because of the Coronavirus pandemic, as an online video-conferencing platform, Zoom Video's (ticker symbol "ZM") shares skyrocketed. However, investors confused an irrelevant company, Zoom Technologies (ticker symbol "ZOOM"), for Zoom Video and bid up Zoom Technologies' price. The Securities and Exchange Commission (SEC) suspended Zoom Technologies' trading to put a stop to this mispricing.[7] In these examples, the prices were distorted for various reasons and deviated from the efficient price. Stambaugh et al. (2012, 2015) construct a very effective mispricing measure based on 11 return anomalies previously reported in the literature, offering empirical support to the institutional foundation of the existence of mispricing for our theoretical model.

Our paper is related to the efficient market hypothesis (EMH) and behavioral models of strategic decision making (Barberis et al. 1998, Fama 1998, Crawford 2013, Harstad and Selten 2013, Rabin 2013). Although the EMH is a useful modeling tool, previous research indicates both theoretically and empirically that perfectly informationally efficient markets cannot exist (Grossman and Stiglitz 1980, Shiller 2003).

Shefrin (2008) proposes three different definitions for market efficiency. The first is predicated on the absence of risk-free arbitrage opportunities. The second hinges on the absence of risky arbitrage opportunities. The third requires prices to mirror fundamental values. There is an abundance of empirical research on market imperfections, yet theoretical studies are sparse.

We can classify the prior theoretical works into two groups. One pertains broadly to *investor informedness*, and the other concerns *investor emotions*.

In the first strand of literature, analytical models examine rational expectations equilibrium (REE). Grossman and Stiglitz (1980) critique informationally efficient market assumptions and expand the rational expectations model by incorporating an information cost. With this cost, they show that market cannot be efficient, and there must be some mispricing because some traders choose not to be informed. Later models support this result and demonstrate that uninformed noise traders can induce severe price volatility (Summers 1986, Cutler et al. 1990, Campbell and Kyle 1993). Brunnermeier (2001) offers a comprehensive survey of asset pricing models under asymmetric information. In this literature, mispricing arises as a result of noise trading (de Long et al. 1990, Madhavan and Smidt 1993, Guasoni 2006).

The second strand of literature is related to behavioral finance and argues that irrational behavior is the source of mispricing. For example, Shiller (1981) and Leroy and Porter (1981) develop models of mispricing in the form of discrete time fads to argue that high volatility can exist even in efficient markets. Many later behavioral finance models are built on top of this logic to explain stock price variations (e.g., West 1988). Barberis et al. (1998), Daniel et al. (1998), and Odean (1998) develop theories of investor overconfidence. Such behavioral biases lead to investor underreaction (overreaction) to information (news), thus resulting in market inefficiency.

Our model is different from these prior theoretical models in two very important ways: First, in our model, mispricing is a cause, not an effect. We examine the situation when the market has already deviated from efficiency and study the outcome of the market. Second, in our model, mispricing is not a binary outcome, but a measure of the extent to which price deviates from what is implied by market efficiency.

We consider a level of price deviation from market efficiency in a dynamic, continuous-time model, in the form of underpricing or overpricing. The equilibrium therefore does not rely on the commonly imposed assumption of semi-strong market efficiency. In the model, different investors have access to different levels of information: An informed trader receives a signal of the liquidation value of the asset and adopts a rational approach to maximize her profit. Algorithm trading adopts feedback-trading strategies and trades on past price changes.[8]

In this study, we focus on one particular type of algorithm trading: feedback trading strategies that are based on past prices.[9] Trend following leads to positive feedback, and contrarian strategies lead to negative feedback (Park and Sabourian 2011). In various financial markets, both institutional and retail investors embrace algorithmic trading tools. For example, recent research has shown that institutional investors' algorithmic trading is associated with increased sensitivity of orders to past returns (Chordia et al. 2008). For completely uninformed investors, Rossi and Tinn (2014) argue that pure price-based trading can have positive profits as long as there exists uncertainty in whether a large trader is informed about fundamentals. In practice, websites, such as Quantopian.com, Interactive Brokers, and mobile apps, such as Robinhood and E*Trade Mobile, make it extremely easy for anyone with minimal financial or programming knowledge to develop price-based strategies and participate in the stock market.

Our model offers insights for two groups of stakeholders: an informed trader and the policy maker. First, for the *informed trader*, our model suggests that the private information on liquidation value will always be realized at market clearance and that the strategy should not be influenced by either mispricing or the intensity of feedback trading. Market mispricing, however, creates profitable opportunities for the informed trader. As long as there exists market mispricing at any time, the informed trader always obtains higher profit than in the case of an efficient market. Second, for the *policy maker*, our results suggest that (1) no matter how price deviates from the efficient level, it will return to the liquidation value at the time of clearance. (2) Not all algorithm traders are bad for the market. Algorithm trading is a double-edged sword. It may reduce the bias to make the market more efficient or increase the bias and lead to even larger price deviation. Algorithm trading that reduces pricing bias is profitable. At the same time, algorithm trading that enlarges pricing bias will lose money. (3) When the asset is overpriced, even some reasonable level of feedback trading that enlarges mispricing can induce a price that is hypersensitive to trading volume, therefore leading to arbitrarily large price movements in the form of bubbles and crashes.

We contribute to the literature in several ways: (1) Methodologically, we formulate a description of a market that is not always efficient. Without such an assumption, traditional models cannot be used to examine the profitability of algorithm trading. This opens doors for future work to examine other aspects of the financial market without the market efficiency assumption. (2) Theoretically, this study generates analytical results related to the formation of bubbles and crashes and derives theoretical implications on when algorithm trading can be profitable.

The remainder of this paper is organized as follows. Section 2 presents the theoretical model, with which we study the impact of market imperfections in the form of initial mispricing. Section 3 provides detailed equilibrium results for this economy, and we investigate the intensity of informed trading and the resultant market depth. Section 4 examines equilibrium price and studies the effects of mispricing and feedback trading. Section 5 examines how the profit gets contributed and redistributed by various parties. Finally, Section 6 concludes.

2. The Model

2.1. The Setup

We consider an informed-trading framework with a continuous-time game. There are three types of traders for an asset on the time horizon $t \in [0, 1]$: (1) a single, rational, risk-neutral informed trader; (2) a representative algorithmic trader,[10] whose orders are composed of both a random noise term (the noise-trading component) and the past prices of the risky asset (the feedback-trading component);[11] and (3) competitive market makers, who set prices based on their observations on the aggregated orders in the market. The informed trader has unique access to the *ex post* liquidation value of the risky asset. The signal is the realization of a random variable, $\tilde{v}$, which is assumed to follow a normal distribution with a mean zero and a variance σ_v^2.[12] The market makers know the distribution of this random variable, but do not know its realization. The representative algorithmic trader does not have information on the distribution of the random variable. The feedback trading strategy depends on past price changes. This model setup is consistent with Kyle (1985), Glosten and Milgrom (1985), de Long et al. (1990), Hong and Stein (1999), and Zhang and Zhang (2015), among others. For easy reference, Table 1 contains the variable definitions.

Table 1. Variable Definitions

Variable	Definition
t	Continuous time. The market starts at $t = 0$ and clears at $t = 1$.
$\tilde{v}$	Exogenous liquidation value of the asset. It is assumed to be normally distributed with mean zero and variance σ_v^2.
$\mathcal{F}_t$	Available information to market makers up to time t.
σ_v^2	Exogenous variance (precision) of the liquidation value $\tilde{v}$.
σ_t^2	Exogenous level of noise trading at time t.
β_t	Exogenous instantaneous feedback trading intensity at time t.
$\tilde{\beta}(t)$	Cumulative feedback trading intensity during time period $[0,t]$. $\tilde{\beta}(t) = \int_0^t \beta_s ds$.
$\overline{\beta}(t)$	Average feedback trading intensity during time period $[0,t]$. $\overline{\beta}(t) = \frac{1}{t}\tilde{\beta}(t) = \frac{1}{t}\int_0^t \beta_s ds$. We denote $\overline{\beta} \equiv \tilde{\beta}(1) = \int_0^1 \beta_s ds$ as the overall feedback trading intensity during the whole duration $t \in [0,1]$.
$\theta(\cdot,\cdot)$	Market deviation reflected on price at time t. It is a continuous function of time t and $\tilde{\beta}_t$, the cumulative intensity of feedback trading from time 0 to t. Endogenously determined.
ε	Initial mispricing. It is given by $\varepsilon = \frac{1}{\theta(0,0)} - 1$, with $\varepsilon \in (-1, \infty)$. It characterizes market imperfections that are independent of feedback trading.
W_t	A one-dimensional standard Brownian motion. $\sigma_t dW_t$ indicates the noise introduced by uninformed traders.
$dX_U(t)$	Order submitted by the uninformed traders at time t.
$dX_I(t)$	Order submitted by the informed trader at time t.
P_t	Price at time t. Endogenously determined.
$\delta(t)$	A measure of the deviation of $V_t = E[\tilde{v}\,|\,\mathcal{F}_t]$ from the liquidation value $\tilde{v}$ up to time t. Specifically, $\delta(t) = E[(\tilde{v} - V_t)^2 \,|\, \mathcal{F}_t]$. It is a very important measure of how much and how fast information gets incorporated into the prices. The smaller $\delta(t)$ is, the faster information gets incorporated into prices. Endogenously determined.
$\pi_I(1)$	Informed trader's cumulative profit up to market clearance. Endogenously determined.
α_t	Informed trader's aggressiveness in adjusting in response to the gap between price and liquidation value. Endogenously determined.
λ_t	The change in price as a result of one unit of increase in total demand at time t. It is a measure of how sensitive the market price is with respect to demand. It is related to the concept of market depth, which is defined as $1/\lambda_t$. Endogenously determined.
ϕ	Exogenous feedback trading's impact on mispricing.

Competitive market makers set prices to clear the market. All uncertainty in the model is supported on a standard probability space $(\Omega, \mathcal{F}, P)$. At any time $t \in [0,1)$, trading occurs in two steps. In the first step, the informed trader and the algorithm trader submit market orders by simultaneously choosing the quantities they will trade.[13] When making the decision, the informed trader's information consists of the private signal of the asset's liquidation value and past prices. The informed trader does not observe current or future asset prices or the past, current, or future quantities traded by the algorithm trader. Within this information environment, the informed trader adopts a trading strategy to maximize her profit. The algorithm trader cannot observe the liquidation value of the risky asset. Similar to the model in Kyle (1985) and Zhang and Zhang (2015), there is a random noise term in their orders. Given the vast number of potential strategies, we do not prescribe the feedback trading as either positive or negative.[14] In the second step, the market makers set the price to clear the market.

The market is closed at time $t = 1$. Denoting the market clearing price as P_1, we allow this price to be potentially different from the expected liquidation value of the asset, $\tilde{v}$. That is, this formulation allows the possibility that the algorithm trader can dominate the market and distort the price, even at the time of market clearance. We later show, analytically, that P_1 converges to the liquidation value of the asset $\tilde{v}$.

2.2. The Informed Trader's Order

We denote the order of the informed trader at time t as $dX_I(t)$. The informed trader's cumulative profit at market clearance can be written as

$$\pi_I(1) = \int_0^1 (P_1 - P_s) dX_I(s). \tag{1}$$

This formulation departs from Kyle (1985) and Zhang and Zhang (2015), as P_1 is not *ex ante* guaranteed to be equal to the liquidation value $\tilde{v}$.

At any time t, the informed trader adjusts her order according to the difference between the current price P_t and her private signal on the liquidation value $\tilde{v}$. The informed order is assumed as

$$dX_I(t) = \alpha_t(\tilde{v} - P_t)dt, \tag{2}$$

where $\alpha_t > 0$ is to be endogenously determined by the informed trader's profit maximization problem. A greater α_t is associated with a more aggressive adjustment in response to the gap between the current price and the liquidation value $\tilde{v}$. The informed trader can realize that the price may not converge to $\tilde{v}$, but because

P_1 is not *ex ante* known, the informed trader chooses her strategy based on $\tilde{v}$.

2.3. Uninformed Traders' Orders

The aggregate order of the algorithm traders at time t can be considered as

$$dX_U(t) = \beta_t dP_{t-} + \sigma_t dW_t, \tag{3}$$

where dP_{t-} denotes recent changes in price just before time t, β_t is the instantaneous aggregated feedback intensity at time t, W_t is a one-dimensional standard Brownian motion, and σ_t is a scaling factor that describes the magnitude of noise trading at time t.[15] The second term on the right-hand side of the equation represents noise trading of uninformed traders. The first term models algorithmic feedback trading.

The feedback intensity, β_t, is aggregately determined by the algorithm traders' strategies. It may vary with respect to time t and can take a positive or negative value. There are a few reasons behind the exogeneity of this parameter: First, each feedback trader can take a potentially different strategy, and there is no way for us to determine the individual feedback intensity of each market participant. The Securities and Exchange Commission released a report and broadly classified four types of strategies: passive market making, arbitrage, structural, and directional (Securities and Exchange Commission 2010). These strategies may rely on different sources of information, may have different ways to react to past prices, and may have different trading objectives. Second, because the traders vary in their capability, many of the strategies are not optimal, and we cannot determine the optimal feedback intensity with a rational expectations framework. Third, the individual strategies may be constantly changing. So, feedback intensity of each trader is a function of previous price changes and time. Overall, it is impossible for us researchers to determine all strategies for all participants in the market. If this variable were to be endogenized, we would have to make very strong and undesirable assumptions on feedback traders' specific strategies. But the problem with such an approach is that β_t is not optimal and should not be modeled with learning algorithms. There are at least two different reasons: (1) it is an aggregated measure with many individual feedback strategies, many of which are not optimal themselves; and (2) the objective of the paper is to examine how the market changes with respect to changes in β_t. So, it is an independent variable and should not be endogenized. This way of modeling also leaves the maximum flexibility for the model to examine the comparative statics associated with β_t. In other words, the exogeneity and unpredictability assumption makes this variable more realistic.

When β_t is positive (negative), the algorithm traders aggregately play a positive (negative) feedback strategy at time t. Based on the instantaneous feedback intensity, we can define the cumulative feedback intensity up to time t as $\tilde{\beta}(t) \equiv \int_0^t \beta_s ds$ and the average feedback intensity up to time t as $\overline{\beta}(t) \equiv \frac{1}{t}\tilde{\beta}(t) = \frac{1}{t}\int_0^t \beta_s ds$. We also denote $\overline{\beta} \equiv \overline{\beta}(1) = \tilde{\beta}(1) = \int_0^1 \beta_s ds$ as the overall feedback trading intensity during the whole duration $t \in [0, 1]$.

There can exist many algorithm traders, each taking a different strategy. In Zhang and Zhang (2015), equations (1) and (2) show that such orders are additive and can be aggregated; therefore, the aggregated orders from all unformed traders can be regarded as one order submitted by a representative algorithmic trader in the model.

2.4. Mispricing

The pricing rule of the market makers can be described by

$$dP_t = \lambda_t[dX_I(t) + dX_U(t)], \tag{4}$$

where $\lambda_t > 0$ will be determined in the equilibrium. Equation (4) describes the market response to aggregated demand and determines how fast the information can be incorporated into the prices. The parameter λ_t is a measure of market depth.[16] Market depth determines the size of the order to move the price by one dollar. When λ_t is large, the market is considered to be shallow, and the price is sensitive to new orders. When λ_t is small in magnitude, the market is deep.

As a reference, we first consider an efficient market in the semi-strong sense. Price at time t should perfectly reflect information available up to that time, and price should take the form:

$$P_t^{efficient} = E[\tilde{v} | \mathcal{F}_t],$$

where $\{\mathcal{F}_t\}_{0 \le t \le 1}$ represents the information available to the market makers up to time t.[17]

In this study, we explicitly consider the existence of a certain market mispricing. We consider the following form of the price:

$$P_t = E[\tilde{v} | \mathcal{F}_t]\theta(\cdot, \cdot), \tag{5}$$

where $\theta(\cdot, \cdot)$ is a continuous function of time t, and $\tilde{\beta}(t)$, the cumulative intensity of feedback trading from time 0 to t. It will be endogenously determined from the equilibrium.[18]

The function $\theta(\cdot, \cdot)$ is a mapping, from (1) time t and (2) cumulative feedback trading up to time t, to a scale factor that moves price.[19] It enlarges or shrinks information available up to time t. It represents a deviation from market efficiency in price. If $\theta(t, \tilde{\beta}(t)) = 1$, it means that, up to time t, price at t perfectly reflects the information available up to t. Whenever $\theta(t, \tilde{\beta}(t)) \neq 1$, there exists mispricing. Specifically, if $\theta(0, 0) \neq 1$, then it means that the initial price does not accurately reflect perfect information available to the market up to time 0.

Equation (5) is the simplest form that captures both the information from liquidation value and the potential impacts from mispricing and algorithm trading at time $t \in [0, 1]$. It allows us to derive closed-form solutions that offer insights into how the algorithm trader affects the equilibrium price of a risky asset, when the algorithm trader makes money in the market, as well as the interactions between the informed trader and the algorithm trader.

Our framework allows us to examine the process of price discovery. The initial mispricing can be understood from a few different perspectives. First, it can represent initial public offering (IPO) underpricing or overpricing (Loughran et al. 1994, Ritter and Welch 2002). IPO underpricing and overpricing refer to the increase or decrease in stock value from the initial offering price to the first-day closing price. This literature suggests that the IPO price can deviate substantially from the fundamentals on the first day of trading. Second, it is possible that the mispricing is not from the very beginning. If the market remains efficient until a certain time, then we can set that time as the beginning of market deviation and let $t = 0$ at that point of time. Third, similarly, the market clearance does not necessarily mean the end of the time when the stock is delisted. It can be anytime when the market deviation is set back to the efficient level, and we can set $t = 1$ at that point of time. Overall, this paper aims to study, for $t \in [0, 1]$, a complete cycle of how market deviation gets corrected.

This form builds a bridge to relate the clearing price, P_1, and the liquidation value, $\tilde{v}$: When the market is closed at time $t = 1$, $P_1 = \tilde{v} \cdot \theta(1, \overline{\beta})$. The clearing price is equal to the liquidation value if and only if the effect of cumulative impact of feedback trading over the whole period satisfies $\theta(1, \overline{\beta}) = 1$ for any $\overline{\beta}$.

Based on this formulation, now, we introduce a market deviation parameter, ε, to describe market initial imperfections that are independent of feedback trading.

This parameter gives us the level of the initial mispricing at time 0. That is,

$$\theta(0, 0) = \frac{1}{1 + \varepsilon}, \tag{6}$$

where $\varepsilon \in (-1, \infty)$; ε can characterize the initial deviation from market efficiency. When $\varepsilon = 0$, we have $\theta(0, 0) = 1$; then, the market is initially semi-strong efficient. When $\varepsilon \neq 0$, the market is either underpriced ($\varepsilon > 0$) or overpriced ($\varepsilon < 0$). No arbitrage implies $\theta(\cdot, \cdot) > 0$ and $\varepsilon > -1$.

3. The Equilibrium

In this section, we first examine the informed trader's strategy and then derive the closed-form solution of the deviation function $\theta(\cdot, \cdot)$.

Theorem 1. *In equilibrium, the informed trader's strategy is*

$$\alpha_t = \frac{\sigma_t}{\sigma_v(1 - t)}.$$

Proof. All proofs are in the online appendix.

Interestingly, the informed trader's equilibrium strategy depends on neither feedback trading nor market mispricing.

The intuition of this result can be understood from the information set available to the informed trader. The informed trader does not know the exact intensity of feedback trading, and she does not need to learn about the deviation. In this situation, her best strategy is to choose α_t to maximize her overall profit, given her knowledge of the liquidation value. This result is consistent with Zhang and Zhang (2015).

We next turn to examine the deviation function $\theta(\cdot, \cdot)$. In the next theorem, we first obtain the general functional form that $\theta(\cdot, \cdot)$ must satisfy, and then we discuss a special case examining the average feedback intensity in the time interval of trading.

Theorem 2. *Function* $\theta(\cdot, \cdot)$ *satisfies*

$$\theta\left(t, \int_0^t \beta_s ds\right) = \frac{1}{1 + \varepsilon(1 - t)\phi^{\overline{\beta}(t)}}, \tag{7}$$

where ϕ *measures the impact of positive feedback trading on mispricing.* ϕ *is a strictly positive constant.*

This theorem gives us a number of important results.[20]

First, when the initial mispricing is zero, then time and level of feedback trading do not influence price deviation, and we always have $\theta(\cdot, \cdot) = 1$. This result suggests that if the market is initially semi-strong efficient, then algorithm trading does not affect the price process in any way, and the price perfectly reflects all available information at any time t.

Second, no matter how feedback trading influences price, time t has the capability of fixing the deviation: When t is close to one, the deviation always disappears. Equation (5) therefore suggests that at the time of market clearance (i.e., $t = 1$), $P_1 = \tilde{v}$ for all β and ε, which implies that no matter how aggressive the algorithm traders are and how severely the market initially deviates from the efficient market, the final price always converges to the liquidation value $\tilde{v}$. Note that $P_1 = \tilde{v}$ does not rely on any particular feedback trading intensity. At the same time, it does not guarantee that the price always perfectly reflects all available information in the market at any time $t < 1$. The convergence is driven by the insider, and it happens with or without feedback trading.

Third, feedback trading's effect on price only depends on the existence of initial mispricing ε. If $\varepsilon = 0$, then the market always remains efficient. At any point of time t,

- When $\phi^{\overline{\beta}(t)} < 1$—that is, in the cases of (1) positive feedback trading and $\phi < 1$ or (2) negative feedback trading and $\phi > 1$—market efficiency is increased with mispricing's effect reduced;
- When $\phi^{\overline{\beta}(t)} > 1$—that is, in the cases of (1) $\phi > 1$ and positive feedback trading or (2) $\phi < 1$ and negative feedback trading—market efficiency is decreased with mispricing's effect enlarged;

- When $\phi^{\overline{\beta}(t)} = 1$—that is, in the case of $\phi = 1$, feedback trading has no impact on mispricing and does not influence market efficiency. In this paper, we do not consider this trivial case.

For simplicity, in the following part of this section, we only consider the case of $\phi < 1$.[21]

Next, we derive results related to market depth. By using Equation (21) from the Appendix, we can derive

$$\lambda_t = \frac{\sigma_v \theta(t, \tilde{\beta}(t))}{\sigma_t + \sigma_v \beta_t \theta(t, \tilde{\beta}(t))},$$

and plugging in Equation (7), we obtain the next theorem.

Theorem 3. *Let $\overline{\beta}(t)$ be the average feedback intensity during time period $[0, t]$ and β_t be the instantaneous feedback intensity at time t; then, we have*

$$\lambda_t = \frac{\sigma_v}{\sigma_v \beta_t + \sigma_t[1 + \varepsilon(1-t)\phi^{\overline{\beta}(t)}]}. \tag{8}$$

For any initial mispricing ε, λ_t is a decreasing function with respect to the instantaneous feedback intensity β_t. It decreases from infinity to $\frac{\sigma_v}{\sigma_t[1+\varepsilon(1-t)\phi^{\overline{\beta}(t)}]}$ and to zero, as β_t increases from a negative value (that makes the denominator be zero) to zero, then to positive infinity.

When $\varepsilon > 0$ ($\varepsilon < 0$), λ_t is an increasing (decreasing) function with respect to the average feedback intensity $\overline{\beta}(t)$. It increases (decreases) from zero (positive infinity) to $\frac{\sigma_v}{\sigma_v\beta_t+\sigma_t[1+\varepsilon(1-t)]}$ and to $\frac{\sigma_v}{\sigma_v\beta_t+\sigma_t}$, as $\overline{\beta}(t)$ increases from a negative value to zero, then to positive infinity.

λ_t is a decreasing function with respect to ε; it decreases from the left side of $\frac{1}{\beta_t}$ to $\frac{\sigma_v}{\sigma_v\beta_t+\sigma_t}$, and to zero, as ε increases from -1 to zero, and to positive infinity.

Interestingly, the instantaneous feedback intensity β_t influences the λ_t directly. However, the influence of initial mispricing ε on λ_t must be through the average feedback trading intensity $\overline{\beta}(t)$ up to time t. In other words, even if there is no initial mispricing, the market depth is affected by the instantaneous feedback intensity β_t. But, if there does not exist initial mispricing, then the average feedback intensity $\overline{\beta}(t)$ over the period from zero to t has no impact on market depth.

Market depth is an important measure of liquidity. A liquid market is generally more efficient. Therefore, the results of Theorem 3 offer insights on how initial mispricing and algorithm trading influence market efficiency. When $\phi < 1$, for any level of initial mispricing (including the case where there is no mispricing), the market becomes more liquid and more efficient with more positive instantaneous feedback trading. In general, positive instantaneous feedback trading increases market efficiency, and negative instantaneous feedback trading reduces market efficiency. This analytical result is consistent with the empirical findings of Hendershott et al. (2011).

The effect of average feedback intensity, on the other hand, depends on the existence of initial mispricing. If there is no initial mispricing, it cannot have an impact. When the market is overpriced (i.e., when $-1 < \varepsilon < 0$), the larger the initial mispricing (more negative ε), the less liquid the market. When the market is underpriced (i.e., when $\varepsilon > 0$), the larger the initial mispricing, the more liquid the market. Average feedback trading weakens both effects, and its impact becomes less significant when the time is close to market clearance. The intuition behind these results is that positive feedback trading can mitigate the effect of initial mispricing and speed up the convergence of the price to the liquidation value. Consequently, among the various algorithm traders, those who follow positive trading strategies, on average, are providing a service to the market to make it more efficient and more liquid.

We examine more detailed dynamics by looking at four special cases.

1. **Case 1:** Semi-Strong Efficient Market (Figure 1)

If the market is initially semi-strong efficient (i.e., if $\varepsilon = 0$), then

$$\lambda_t = \frac{\sigma_v}{\sigma_v \beta_t + \sigma_t} = \frac{1}{\beta_t + \frac{\sigma_t}{\sigma_v}}.$$

The effect of the average feedback trading intensity $\overline{\beta}(t)$ disappears when there is no initial mispricing. However, the instantaneous feedback intensity β_t still affects λ_t.

This result suggests that the price stability at time t depends on (1) the instantaneous feedback trading's intensity and (2) the relative strength of the two variance measures. The higher the feedback trading at current time t, the deeper the market. That is, higher instantaneous feedback trading intensity leads to a more stable market. When the level of noise trading (σ_t) is high or when the uncertainty of the signal (σ_v) is low, the market is deeper for the same level of feedback trading. Intuitively, when the market is characterized by a higher level of noise trading or by a lower level of uncertainty of the liquidation value, the market is deeper and can accommodate higher levels of negative feedback trading.

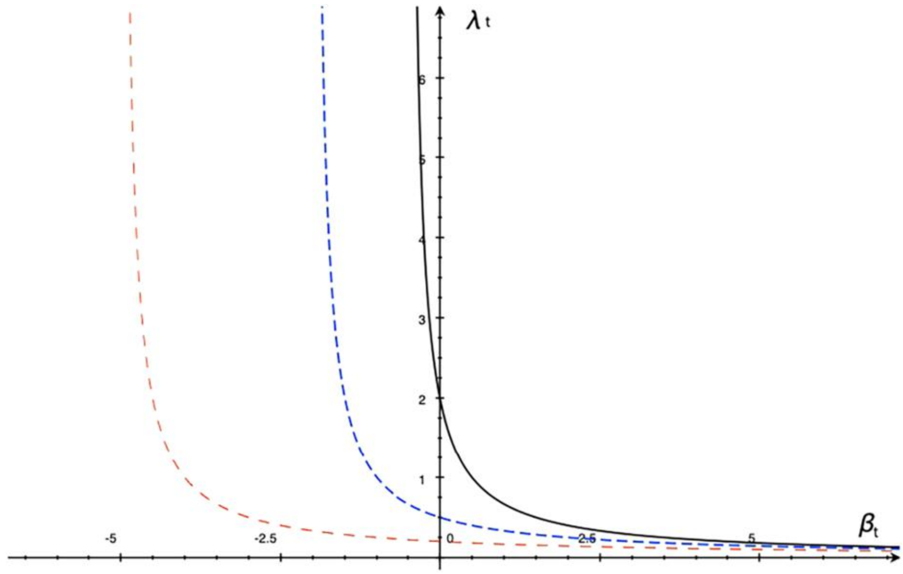
2. **Case 2:** Effect of Average Feedback (Figure 2)

In order to investigate the effect of average feedback only, we assume that the instantaneous feedback intensity $\beta_t = 0$, but the average feedback intensity $\overline{\beta}(t)$ during time period $[0, t]$ is not zero, then

$$\lambda_t = \frac{\sigma_v}{\sigma_t[1 + \varepsilon(1-t)\phi^{\overline{\beta}(t)}]} = \frac{1}{\frac{\sigma_t}{\sigma_v}[1 + \varepsilon(1-t)\phi^{\overline{\beta}(t)}]}.$$

This suggests that the average feedback intensity $\overline{\beta}(t)$ during the time period $[0, t]$ affects λ_t only through initial mispricing. Similar to the first case and intuitively,

Figure 1. (Color online) Case 1: Semi-Strong Efficient Market

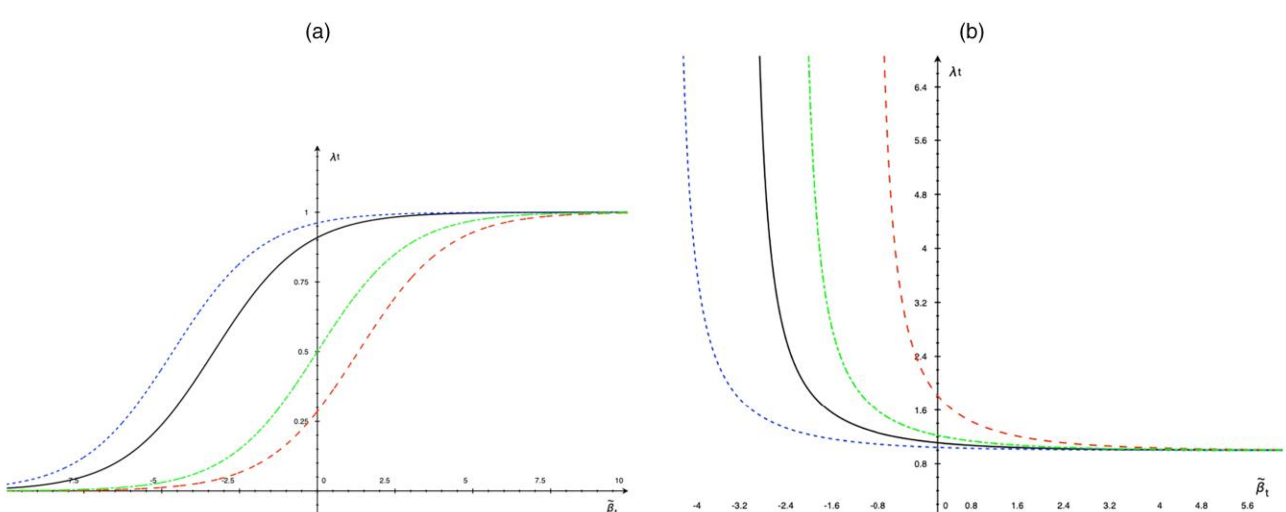


Notes. Solid line: $\frac{\sigma_t}{\sigma_v} = \frac{1}{2}$. Dotted line: $\frac{\sigma_t}{\sigma_v} = 2$. Dashed line: $\frac{\sigma_t}{\sigma_v} = 5$.

the market becomes deeper when there is a higher level of noise trading or a lower level of uncertainty of the liquidation value.

- When the market is initially underpriced (i.e., $\varepsilon > 0$), as $\overline{\beta}(t)$ increases from negative infinite to zero, and to positive infinity, λ_t increases from zero to $\frac{\sigma_v}{\sigma_t[1+\varepsilon]}$, and to $\frac{\sigma_v}{\sigma_t}$. Figure 2(a) shows that market depth reduces with the average feedback intensity in general. When it is closer to market clearance, the price reacts to orders more aggressively. Higher initial underpricing is associated with a deeper market. Negative feedback trading, in general, leads to a deeper market. In this case, because λ_t is bounded between zero and $\frac{\sigma_v}{\sigma_t}$, the price cannot vary too much, and feedback trading cannot lead to any bubbles and crashes.
- For $\varepsilon < 0$—that is, when the market is initially overpriced—for any specific t and ε, there exists a negative $\overline{\beta}(t)$, such that λ_t equals positive infinity. Therefore, when $\overline{\beta}(t)$ increases from this

Figure 2. (Color online) Case 2: Effect of Cumulative Feedback



Notes. (a) Underpricing. (b) Overpricing. (a) Solid line: moderate underpricing and far from market clearance with $\frac{\sigma_t}{\sigma_v} = 1$, $\varepsilon = 0.2$, and $t = 0.5$. Dotted line: moderate underpricing and closer to market clearance with $\frac{\sigma_t}{\sigma_v} = 1$, $\varepsilon = 0.2$, and $t = 0.8$. Dashed line: higher underpricing and far from market clearance with $\frac{\sigma_t}{\sigma_v} = 1$, $\varepsilon = 5$, and $t = 0.5$. Dashdot line: higher underpricing and closer to market clearance with $\frac{\sigma_t}{\sigma_v} = 1$, $\varepsilon = 5$, and $t = 0.8$. (b) Solid line: moderate overpricing and far from market clearance with $\frac{\sigma_t}{\sigma_v} = 1$, $\varepsilon = -0.2$, and $t = 0.5$. Dotted line: moderate overpricing and closer to market clearance with $\frac{\sigma_t}{\sigma_v} = 1$, $\varepsilon = -0.2$, and $t = 0.8$. Dashed line: higher overpricing and far from market clearance with $\frac{\sigma_t}{\sigma_v} = 1$, $\varepsilon = -0.9$, and $t = 0.5$. Dashdot line: higher overpricing and closer to market clearance with $\frac{\sigma_t}{\sigma_v} = 1$, $\varepsilon = -0.9$, and $t = 0.8$.

negative value to zero, and to positive infinity, λ_t decreases from positive infinity to $\frac{\sigma_v}{\sigma_t[1+\varepsilon(1-t)]}$, and to $\frac{\sigma_v}{\sigma_t}$. From Figure 2(b), we can see these results.

3. **Case 3:** Effect of Instantaneous Feedback (Figure 3)

In order to study the effect of instantaneous feedback, we assume that the average feedback intensity $\overline{\beta}(t)$ during the time period $[0, t]$ is zero, but the instantaneous feedback intensity β_t is not zero, then

$$\lambda_t = \frac{\sigma_v}{\sigma_v\beta_t + \sigma_t[1+\varepsilon(1-t)]} = \frac{1}{\beta_t + \frac{\sigma_t}{\sigma_v}[1+\varepsilon(1-t)]}.$$

This case is similar to Case 1, but with an additional term involving initial mispricing. Notice that for any ε, there exists a negative β_t such that $\sigma_v\beta_t + \sigma_t[1+\varepsilon] = 0$. As β_t increases from this value to zero, then to positive infinity, λ_t decreases from positive infinity to $\frac{\sigma_v}{\sigma_t[1+\varepsilon]}$, then to zero. Figure 3 shows that the same level of instantaneous feedback intensity is associated with a deeper (shallower) market when there is initial underpricing (overpricing). A higher level of overpricing is associated with a less tolerant level of feedback trading. In Figure 3, the green line quickly goes to infinity when feedback trading is getting less intensified. In other words, when there is overpricing, feedback trading needs to be positive and maintain a high level. Otherwise, a small change in order can lead to a significant change to price.

4. **Case 4:** Effect of Mispricing (Figure 4)

If there is no feedback trading, then the algorithm trader is just a noise trader (i.e., $\overline{\beta}(t) = \beta_t = 0$), and

$$\lambda_t = \frac{\sigma_v}{\sigma_t[1+\varepsilon(1-t)]} = \frac{1}{\frac{\sigma_t}{\sigma_v}[1+\varepsilon(1-t)]}.$$

The effect of the initial mispricing depends on the relative strength of the two variance measures. Again, the result is different from that of Kyle (1985) and Zhang and Zhang (2015). When $\varepsilon > 0$, the asset is initially underpriced. In this case, initial mispricing increases market depth and, intuitively, makes it more difficult for order flows to move the price. Conversely, when ε moves from zero toward -1, $\theta(t, 0)$ increases to infinity. In this overpriced case, the market becomes shallow, and the price becomes very sensitive to orders. The market can be extremely risky when ε moves close to -1.

As can be seen from Figure 4, when there is no feedback trading, the market depth is affected by (1) the initial mispricing and (2) the relative levels of noise trading and uncertainty of the liquidation value. Underpricing is associated with a more stable market. When ε moves from a positive value to -1, the market becomes thinner and creates opportunities for extreme price movements.

Based on these results, we can see that overpricing has a much more significant and detrimental impact on price than underpricing.

4. Equilibrium Price

In this section, we study the equilibrium price.

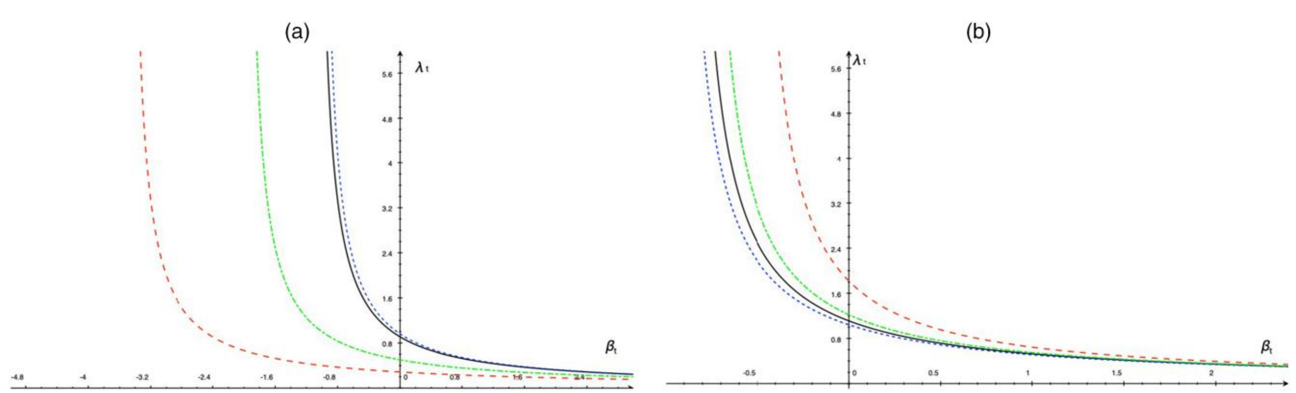
From the proof of Theorem 2, we have the stochastic differential equation for the equilibrium price:

$$dP_t = \frac{V_t}{1-t}\theta(t, \tilde{\beta}(t))[1-\theta(t, \tilde{\beta}(t))]dt + \sigma_v\theta(t, \tilde{\beta}(t))dW_t,$$

where $V_t = E(\tilde{v}|\mathcal{F}_t)$.

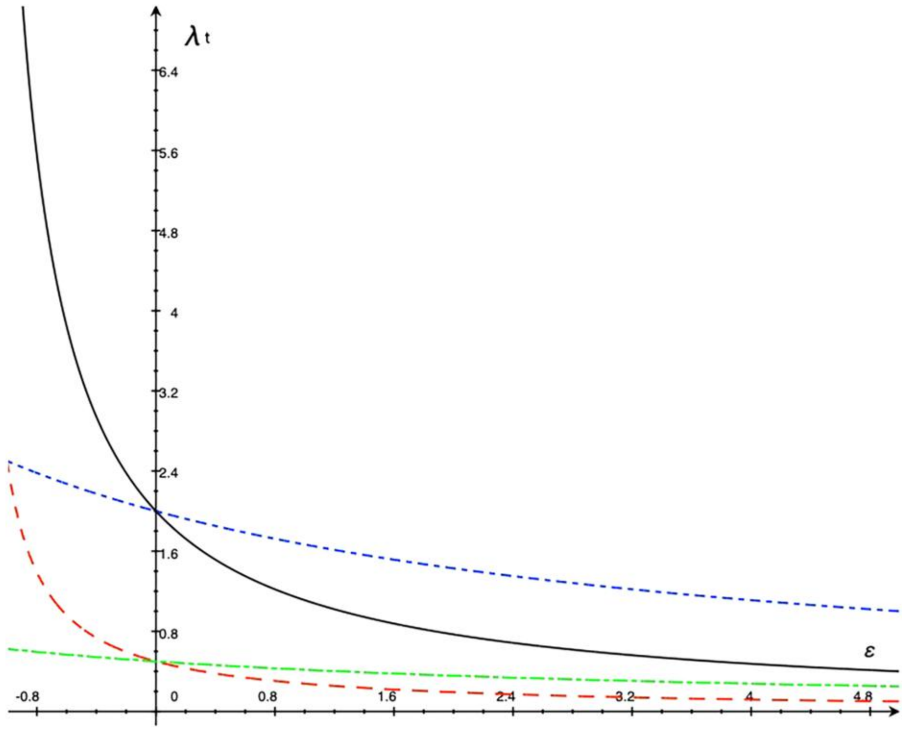
We use $\overline{\beta}(t)$ to represent the average feedback intensity in the time interval $[0, t]$.[22] Then, the above equation

Figure 3. (Color online) Case 3: Effect of Instantaneous Feedback



Notes. (a) Underpricing. (b) Overpricing. (a) Solid line: moderate underpricing and far from market clearance with $\frac{\sigma_t}{\sigma_v} = 1$, $\varepsilon = 0.2$, and $t = 0.5$. Dotted line: moderate underpricing and closer to market clearance with $\frac{\sigma_t}{\sigma_v} = 1$, $\varepsilon = 0.2$, and $t = 0.8$. Dashed line: higher underpricing and far from market clearance with $\frac{\sigma_t}{\sigma_v} = 1$, $\varepsilon = 5$, and $t = 0.5$. Dashdot line: higher underpricing and closer to market clearance with $\frac{\sigma_t}{\sigma_v} = 1$, $\varepsilon = 5$, and $t = 0.8$. (b) Solid line: moderate overpricing and far from market clearance with $\frac{\sigma_t}{\sigma_v} = 1$, $\varepsilon = -0.2$, and $t = 0.5$. Dotted line: moderate overpricing and closer to market clearance with $\frac{\sigma_t}{\sigma_v} = 1$, $\varepsilon = -0.2$, and $t = 0.8$. Dashed line: higher overpricing and far from market clearance with $\frac{\sigma_t}{\sigma_v} = 1$, $\varepsilon = -0.9$, and $t = 0.5$. Dashdot line: higher overpricing and closer to market clearance with $\frac{\sigma_t}{\sigma_v} = 1$, $\varepsilon = -0.9$, and $t = 0.8$.

Figure 4. (Color online) Case 4: Effect of Mispricing



Notes. Solid line: $\frac{\sigma_t}{\sigma_v} = \frac{1}{2}$, $t = 0.2$. Dotted line: $\frac{\sigma_t}{\sigma_v} = \frac{1}{2}$, $t = 0.8$. Dashed line: $\frac{\sigma_t}{\sigma_v} = 2$, $t = 0.2$. Dashdot line: $\frac{\sigma_t}{\sigma_v} = 2$, $t = 0.8$.

can be rewritten as

$$dP_t = \frac{V_t}{1-t} \cdot \frac{\varepsilon(1-t)\phi^{\overline{\beta}(t)}}{[1+\varepsilon(1-t)\phi^{\overline{\beta}(t)}]^2} dt + \frac{\sigma_v}{1+\varepsilon(1-t)\phi^{\overline{\beta}(t)}} dW_t. \tag{9}$$

In this equation, the first term on the right-hand side is a drift term, and the second is a diffusion term.

The price at time t can be given as

$$P_t = V_t \theta(t, \tilde{\beta}(t)) = \frac{V_t}{1+\varepsilon(1-t)\phi^{\overline{\beta}(t)}}. \tag{10}$$

Clearly, the price perfectly reflects the liquidation value with all available information only when ε is zero. Algorithm trading's impact depends on a nonzero ε: When there is no initial mispricing, algorithm trading does not play any role in the price. When initial mispricing exists, regardless of the sign of ε, when $\phi^{\overline{\beta}(t)} < 1$—that is, in the cases of (1) positive feedback trading with $\phi < 1$ or (2) negative feedback trading with $\phi > 1$—algorithm trading reduces initial mispricing's effect on price at time t. That is, $\phi^{\overline{\beta}(t)} < 1$ moves $P_t = \frac{V_t}{1+\varepsilon(1-t)\phi^{\overline{\beta}(t)}}$ closer to V_t than when $\overline{\beta}(t) = 0$. The intuition behind this result is straightforward: when $\phi^{\overline{\beta}(t)} < 1$, algorithm trading plays a role similar to informed trading. No matter whether there is underpricing or overpricing, it will speed up the price correction process. This result offers important policy implications because although algorithm trading may increase the price movement, our result suggests that sometimes it is in the right direction.

When $\phi^{\overline{\beta}(t)} > 1$—that is, in the cases of (1) $\phi > 1$ with positive feedback trading or (2) $\phi < 1$ with negative feedback trading—algorithm trading enlarges the effect of mispricing and, therefore, decreases market efficiency. Depending on the sign of ε, we can examine how the price deviates from predictions of the efficient market hypothesis. There are two cases:

- **Case 1:** Underpricing ($\varepsilon > 0$).

  1. The drift term: From (9), the drift term is strictly positive. Function $\frac{\varepsilon(1-t)\phi^{\overline{\beta}(t)}}{[1+\varepsilon(1-t)\phi^{\overline{\beta}(t)}]^2}$ reaches its maximum value $\frac{1}{4}$ when $\varepsilon(1-t)\phi^{\overline{\beta}(t)} = 1$, further when $\varepsilon(1-t)\phi^{\overline{\beta}(t)} > 1$, $\frac{\varepsilon(1-t)\phi^{\overline{\beta}(t)}}{[1+\varepsilon(1-t)\phi^{\overline{\beta}(t)}]^2}$ monotonically decrease with $\varepsilon(1-t)\phi^{\overline{\beta}(t)}$, and when $\varepsilon(1-t)\phi^{\overline{\beta}(t)} < 1$, $\frac{\varepsilon(1-t)\phi^{\overline{\beta}(t)}}{[1+\varepsilon(1-t)\phi^{\overline{\beta}(t)}]^2}$ monotonically increase with $\varepsilon(1-t)\phi^{\overline{\beta}(t)}$.

  For fixed ε and t, we can have

  – If $\phi < 1$,

$$\lim_{\overline{\beta}(t)\to-\infty} \frac{\varepsilon(1-t)\phi^{\overline{\beta}(t)}}{[1+\varepsilon(1-t)\phi^{\overline{\beta}(t)}]^2} = 0.$$

– If $\phi > 1$,

$$\lim_{\overline{\beta}(t)\to+\infty} \frac{\varepsilon(1-t)\phi^{\overline{\beta}(t)}}{[1+\varepsilon(1-t)\phi^{\overline{\beta}(t)}]^2} = 0.$$

So an increasing $|\overline{\beta}(t)|$ induces a reduction in the drift. Eventually, $\overline{\beta}(t)$'s effect disappears when t approaches one.

2. The diffusion term: Because $\varepsilon(1-t)\phi^{\overline{\beta}(t)} > 0$, the volatility of the price is always smaller than that in a semi-strong efficiency market. For fixed ε and time t,

– If $\phi < 1$,

$$\lim_{\overline{\beta}(t)\to-\infty} \frac{\sigma_v}{1+\varepsilon(1-t)\phi^{\overline{\beta}(t)}} = 0.$$

So a strong negative-feedback algorithm strategy will drive the price volatility to zero.

– If $\phi > 1$,

$$\lim_{\overline{\beta}(t)\to+\infty} \frac{\sigma_v}{1+\varepsilon(1-t)\phi^{\overline{\beta}(t)}} = 0.$$

Again, a strong positive-feedback algorithm strategy will drive the price volatility to zero.

Put together, when the asset is underpriced, algorithm trading is associated with moderate price movements.

- **Case 2:** Overpricing ($\varepsilon < 0$).

First of all, given the sign of ε, we always have $1 + \varepsilon(1-t)\phi^{\overline{\beta}(t)} < 1$. Note the price at time t is $P_t = \frac{V_t}{1+\varepsilon(1-t)\phi^{\overline{\beta}(t)}}$. For no arbitrage, we require the denominator to be positive. So, we always have:

$$0 < 1 + \varepsilon(1-t)\phi^{\overline{\beta}(t)} < 1.$$

Therefore, feedback intensity must satisfy the following relation: $\phi^{\overline{\beta}(t)} < -\frac{1}{\varepsilon(1-t)}$. With this inequality and condition $\phi^{\overline{\beta}(t)} > 1$, we know that

– When $\phi < 1$, average negative feedback intensity should satisfy

$$\frac{\ln\left(\frac{1}{-\varepsilon(1-t)}\right)}{\ln(\phi)} < \overline{\beta}(t) < 0.$$

– When $\phi > 1$, average positive feedback intensity should satisfy

$$0 < \overline{\beta}(t) < \frac{\ln\left(\frac{1}{-\varepsilon(1-t)}\right)}{\ln(\phi)}.$$

Let $\beta^{**} = \frac{\ln\left(\frac{1}{-\varepsilon(1-t)}\right)}{\ln(\phi)}$.

1. The drift term: From (9), the drift term is strictly negative, which implies that the price itself has the tendency to decrease to the efficient level V_t.

It is easy to see that when $1 < \phi^{\overline{\beta}(t)} < -\frac{1}{\varepsilon(1-t)}$,

$$\frac{\varepsilon(1-t)\phi^{\overline{\beta}(t)}}{[1+\varepsilon(1-t)\phi^{\overline{\beta}(t)}]^2} < \frac{\varepsilon(1-t)}{[1+\varepsilon(1-t)]^2} < 0.$$

Again, the effect of algorithm trading depends on ε being nonzero.

2. The diffusion term: For an overpriced asset, the volatility of price is higher than the case with efficient market assumptions.

– When $\phi < 1$ and the average negative feedback $\overline{\beta}(t)$ approaches β^{**} from the right, $\frac{\sigma_v}{1+\varepsilon(1-t)\phi^{\overline{\beta}(t)}}$ goes to infinity. In this case, the volatility of the price can be arbitrarily large, so bubbles and crashes emerge in the market.

– When $\phi > 1$ and the average positive feedback $\overline{\beta}(t)$ approaches β^{**} from the left, $\frac{\sigma_v}{1+\varepsilon(1-t)\phi^{\overline{\beta}(t)}}$ goes to infinity. In this case, the volatility of the price can also be arbitrarily large, so bubbles and crashes emerge in the market.

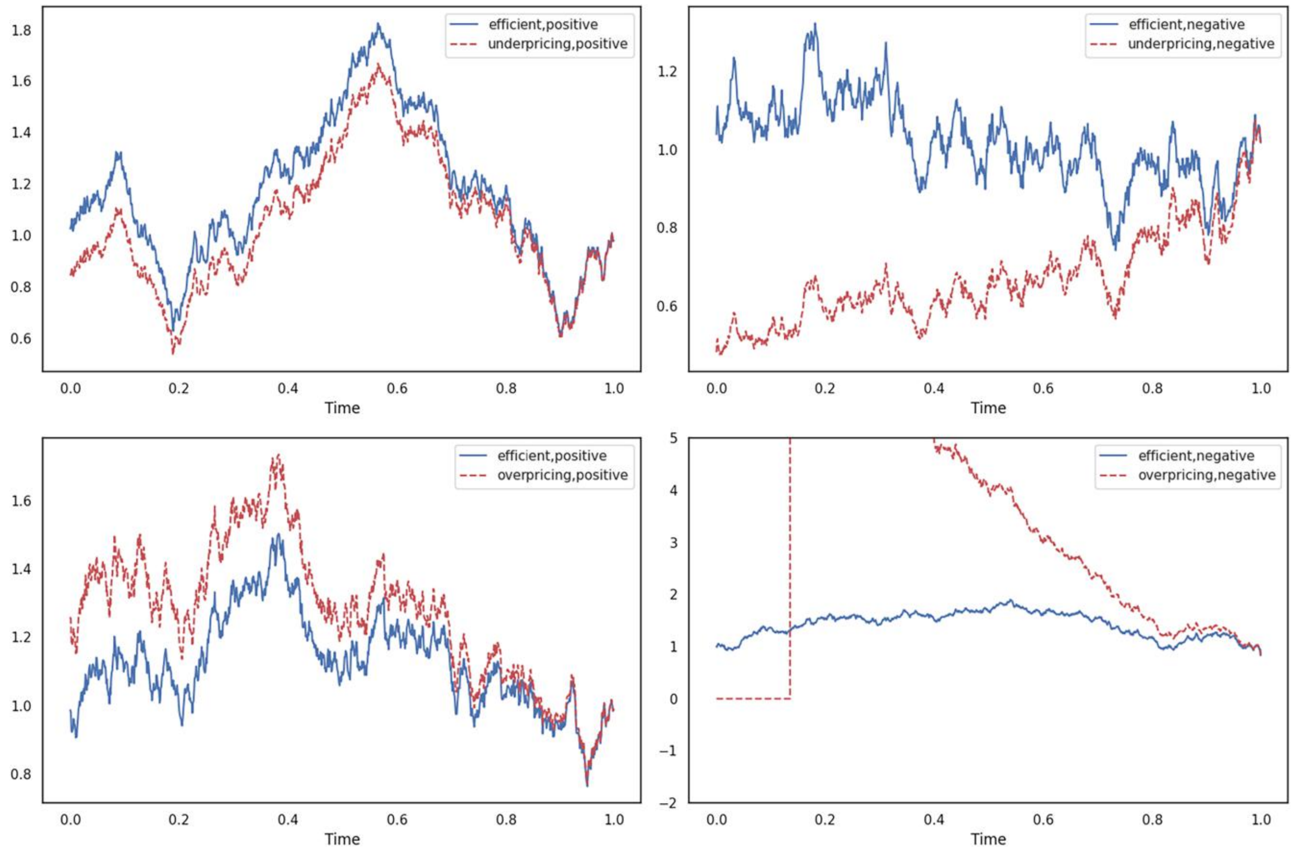
The intuition behind these results related to negative feedback trading can be understood by examining Figures 2 and 4. Compared with underpricing, overpricing has much higher risk of facing the problem of an illiquid market (with λ going to infinity under some conditions). Without liquidity, a small change in price can lead to significant ups and downs of the market. Because the market is generally converging to the efficient level, no matter how large the feedback trading is, any force going against this general trend (negative feedback trading is such a force) has the potential to lead to further market deviation.

We can plot how the price process evolves over time. Equation (10) shows the price as a function of time. We just need to derive V_t as follows:

$$V_t = V_0(1-t) + \tilde{v}\cdot t - (1-t)\int_0^t \frac{\sigma_v}{(1-s)^2}dW_s,$$

where W_s is a standard Brownian motion, and it introduces some randomness to the trajectory. The four panels of Figure 5 show how price converges to the liquidation value when $\phi < 1$:[23] upper left (underpricing, positive feedback trading), upper right (underpricing, negative feedback trading), lower left (overpricing, positive feedback trading), and lower right (overpricing, negative feedback trading). In all panels, the blue lines indicate the efficient cases with $\varepsilon = 0$. In all cases, the price converges to the liquidation value of $\tilde{v} = 1$ at time $t = 1$. With positive feedback trading, price convergence is much faster than the case of negative feedback trading. Negative feedback trading in an overpriced market creates bubbles and crashes, but, eventually, the price still converges to the fundamental value.

Figure 5. (Color online) Price Trajectory



Notes. $\tilde{v} = 1$, $V_0 = 1$, $\sigma_v = 0.6$. Upper left: underpricing and positive feedback with $\varepsilon = 0.5$ and $\overline{\beta}(t) = 1.2$. Upper right: underpricing and negative feedback with $\varepsilon = 0.5$ and $\overline{\beta}(t) = -1.2$. Lower left: overpricing and positive feedback with $\varepsilon = -0.5$ and $\overline{\beta}(t) = 1.2$. Lower right: overpricing and negative feedback with $\varepsilon = -0.5$ and $\overline{\beta}(t) = -1.2$. All solid lines have $\varepsilon = 0$.

Overall, feedback trading that corrects the initial mispricing (i.e., $\phi^{\overline{\beta}(t)} < 1$) can make the market more efficient and facilitate price discovery because it is consistent with insider trading and makes information more transparent. On the other hand, feedback trading goes against insider trading (i.e., $\phi^{\overline{\beta}(t)} > 1$), makes information less transparent, and hinders price discovery, leading to increased deviation of the market away from efficiency.

These results generate important policy implications. Algorithm traders should not be blamed for market deviations. Our results show that only those "bad" traders who cannot correctly determine the price direction are causing trouble, and their impact is only pronounced when there is market overpricing. Therefore, in an effort to protect investors, regulatory authorities should focus more on preventing such "bad" market participants from exacerbating market deviations, rather than regulating the whole market of algorithm traders.

5. Redistribution of Profit

Does algorithm trading increase or decrease the informed trader's profit? When is algorithm trading profitable? In this section, we investigate the profit earned by the informed trader.

Theorem 4. *The expectation of the profit earned by the informed trader during the whole period can be expressed as*

$$E[\pi_I(1)] = \int_0^1 \sigma_t \sigma_v \left[1 + \frac{\varepsilon^2(1-t)^2\phi^{2\overline{\beta}(t)}}{[1+\varepsilon(1-t)\phi^{\overline{\beta}(t)}]^2}\right] dt. \quad (11)$$

We can see that higher levels of noise trading σ_t are associated with higher profit for the insider. At the same time, σ_v, as a measure of the insider's information advantage, is associated with higher insider's profit. We always have

$$\frac{\varepsilon^2 t(1-t)^2\phi^{2\overline{\beta}(t)}}{[1+\varepsilon(1-t)\phi^{\overline{\beta}(t)}]^2} > 0,$$

for all $\overline{\beta}(t)$, $\varepsilon \neq 0$, and $t \in (0,1)$. This implies that initial deviation ε always leads to more profit for the insider, no matter whether this deviation ε is negative or positive.

We can view the profit of the informed trader as a function of ε and β:

$$f(\varepsilon,\beta) \equiv E[\pi_I(1)] = \int_0^1 \sigma_t\sigma_v\left[1+\frac{\varepsilon^2(1-t)^2\phi^{2\overline{\beta}(t)}}{[1+\varepsilon(1-t)\phi^{\overline{\beta}(t)}]^2}\right]dt,$$

where β denotes $\overline{\beta}(t)$, which may vary with time t, and it is not a constant.

Notice that $f(0,\beta)=\int_0^1\sigma_t\sigma_v dt = f(0,0)$ for all β. This is the case in Zhang and Zhang (2015), where there is no initial mispricing, and the market is semi-strong efficient.

Based on Equation (11), we can plot how insider's profit changes over time in Figure 6, where $\phi < 1$.

The profit accumulates over time. In the case of underpricing, the insider earns higher profit if the feedback is negative (dotted line versus solid line). With respect to overpricing, we find that (1) compared with the case of underpricing, the same level of positive feedback gives the insider higher profit (dashed line versus solid line); and (2) the highest insider profit is obtained when the market is overpriced and the feedback traders adopts negative feedback strategies (dashdot line).

5.1. Detailed Properties of the Function $f(\varepsilon,\beta)$

In order to study the different effects of initial mispricing ε and the feedback intensity $\overline{\beta}(t)$, first, we investigate the properties of the function $f(\varepsilon,\beta)$.

1. As a function of ε, $f(\varepsilon,\beta)$ reaches its minimum value

$$f(0,\beta)=f(0,0)=\int_0^1\sigma_t\sigma_v dt$$

at $\varepsilon=0$ for all functions β because

$$\frac{\partial f}{\partial\varepsilon}=\int_0^1\sigma_t\sigma_v\frac{2\varepsilon(1-t)^2\phi^{2\overline{\beta}(t)}}{[1+\varepsilon(1-t)\phi^{\overline{\beta}(t)}]^3}dt;$$

when $\varepsilon>0$, $\frac{\partial f}{\partial\varepsilon}>0$, this implies that for any positive $\varepsilon>0, f(\varepsilon,\beta)>f(0,\beta)$, and when $\varepsilon<0$, $\frac{\partial f}{\partial\varepsilon}<0$, this implies that for any negative $\varepsilon<0, f(\varepsilon,\beta)>f(0,\beta)$.

This shows that the initial market mispricing ε always brings higher profit for the insider than that of a semi-strong efficient market, no matter whether ε is positive or negative. This result is intuitive: market deviation can be understood as trading opportunities arising from the market. The insiders are able to capture these opportunities to increase their profit.

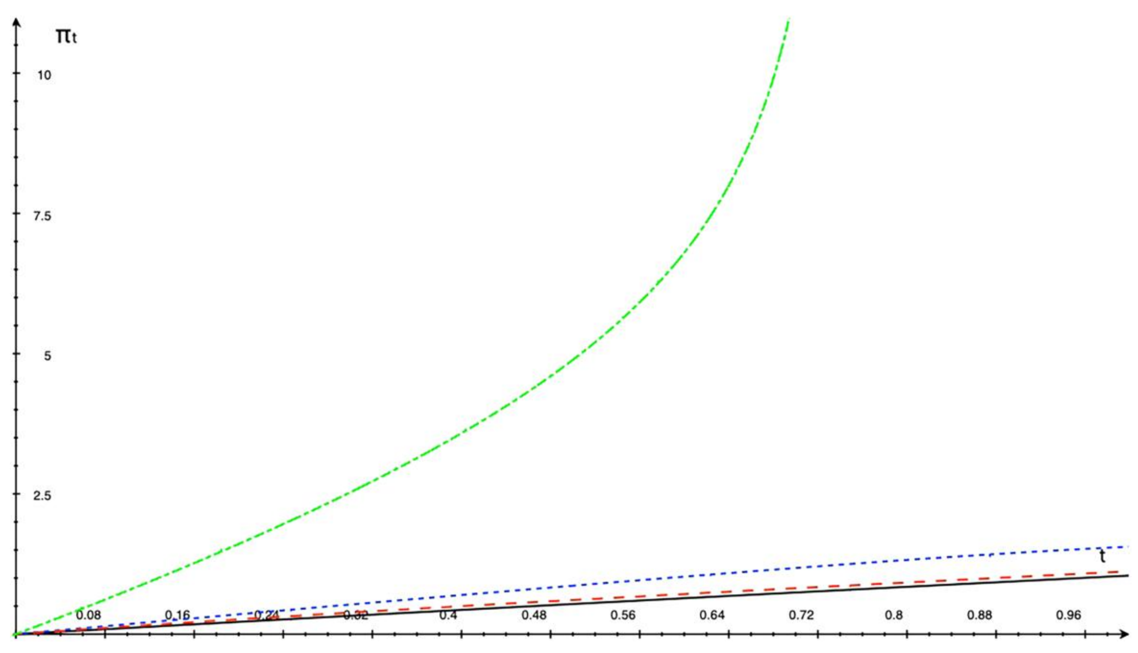
Moreover,

- In case of $\varepsilon>0$,

$$\lim_{\varepsilon\to+\infty} f(\varepsilon,\beta)=2f(0,0).$$

This suggests that, with initial underpricing, the insider's profit as the result of initial mispricing has an upper bound.

Figure 6. (Color online) Insider's Profit as a Function of Time



Notes. Solid line: underpricing and positive feedback with $\varepsilon=2$ and $\overline{\beta}(t)=2$. Dotted line: underpricing and negative feedback with $\varepsilon=2$ and $\overline{\beta}(t)=-2$. Dashed line: overpricing and positive feedback with $\varepsilon=-0.9$ and $\overline{\beta}(t)=2$. Dashdot line: overpricing and negative feedback with $\varepsilon=-0.9$ and $\overline{\beta}(t)=-2$.

- In case of $\varepsilon \in (-1,0)$, under condition $1+\varepsilon(1-t)\phi^{\overline{\beta}(t)} > 0$ for all possible $\overline{\beta}(t)$ and ε, $f(\varepsilon,\beta)$ may explode and can even go to infinity when $\overline{\beta}(t)$ makes $1+\varepsilon(1-t)\phi^{\overline{\beta}(t)}$ approach zero. Combined with the results from the last section, we know that the insider's profit is provided by the feedback traders whose trading strategies enlarge mispricing and make the price deviate more from the semi-strong efficient price. Specifically, when there is overpricing, this feedback trading can destabilize the market and lose a significant amount of money as a punishment by the market.

In summary, as ε goes from -1 to zero and satisfies $1+\varepsilon(1-t)\phi^{\overline{\beta}(t)} > 0$ for all possible $\overline{\beta}(t)$ and $\varepsilon, f(\varepsilon,\beta)$ decreases from positive infinity to $f(0,\beta)$, its minimum value; then, as ε goes from zero to positive infinity, $f(\varepsilon,\beta)$ increases from $f(0,0)$ to $2f(0,0)$.

These results imply that (1) the insider's profit is higher when there exists initial mispricing ε (compared with the case of semi-strong efficient market); and (2) compared with initial underpricing, initial overpricing can bring unbounded insider profit.

2. In order to study the effect of initial mispricing ε, we let $\beta = 0$,[24] at any time $t \in (0,1)$

$$f(\varepsilon,0) = \int_0^1 \sigma_t\sigma_v\left(1+\frac{\varepsilon^2(1-t)^2}{[1+\varepsilon(1-t)]^2}\right)dt$$

$$= f(0,0) + \int_0^1 \frac{\sigma_t\sigma_v\varepsilon^2(1-t)^2}{[1+\varepsilon(1-t)]^2}dt.$$

This is an increasing function of $|\varepsilon|$. As ε goes from -1 to zero, and from zero to positive infinite, $f(\varepsilon,0)$ decreases from positive infinite to $f(0,0)$ and increases from $f(0,0)$ to $2f(0,0)$. This result measures the contribution of pure noise trading to the insider's profit. $f(\varepsilon,0) - f(0,0) = \int_0^1 \frac{\sigma_t\sigma_v\varepsilon^2(1-t)^2}{[1+\varepsilon(1-t)]^2}dt$ is the additional contribution of noise trading to the insider's profit when there is initial mispricing.

3. Note that under condition $1+x > 0$, function $\frac{x^2}{(1+x)^2}$ is decreasing when $x < 0$ and increasing when $x > 0$. Therefore, the minimum value is zero when $x = 0$. Based on this, we have

- When $\phi^{\overline{\beta}(t)} < 1$, for all possible ε,

$$\lim_{\phi^{\overline{\beta}(t)}\to 0} f(\varepsilon,\beta) = f(0,0).$$

This implies that the most aggressive feedback traders who reduce the mispricing and make the pricing close to semi-strong efficient behave similarly as insiders and can earn all profit caused by the initial mispricing.

- When $\phi^{\overline{\beta}(t)} > 1$,
  - When the market is initially underpriced—that is, for positive ε,

$$\lim_{\phi^{\overline{\beta}(t)}\to +\infty} f(\varepsilon,\beta) = 2f(0,0).$$

  - When the market is initially overpriced, with $\varepsilon \in (-1,0)$, under condition

$$1+\varepsilon(1-t)\phi^{\overline{\beta}(t)} > 0$$

for all possible $\overline{\beta}(t)$ and ε, $f(\varepsilon,\beta)$ can be very big and may go to infinity.

In summary, when the market is initially underpriced—that is, for positive ε, $f(\varepsilon,\beta)$ increases as $\phi^{\overline{\beta}(t)}$ increases—this implies that feedback traders who enlarge the mispricing and make the price more deviant from the efficient market price will contribute more to the insider's profit. Specifically, as $\phi^{\overline{\beta}(t)}$ goes from zero to one, then to positive infinity, the profit $f(\varepsilon,\beta)$ increases from $f(0,0)$ to $f(0,0) + \int_0^1 \frac{\sigma_t\sigma_v\varepsilon^2(1-t)^2}{[1+\varepsilon(1-t)]^2}dt$, then to $2f(0,0)$,

In an initial overpriced market, as $\phi^{\overline{\beta}(t)}$ goes from zero to one, then to $-\frac{1}{\varepsilon(1-t)}$ from the left side, $f(\varepsilon,\beta)$ increases from $f(0,0)$, to $f(0,0) + \int_0^1 \frac{\sigma_t\sigma_v\varepsilon^2(1-t)^2}{[1+\varepsilon(1-t)]^2}dt$, then to positive infinity. This implies that the feedback traders who enlarge the mispricing and make the price more deviant from the efficient market price will contribute infinity to the insider's profit.

5.2. Profit Decomposition

With the above results, we can further decompose the profit earned by the insider as

$$E[\pi_I(1)] = f(0,0) + [f(\varepsilon,0) - f(0,0)] + [f(\varepsilon,\beta) - f(\varepsilon,0)],$$

where the first term $f(0,0)$ is the profit earned by the insider from both feedback trading and noise trading when the market is semi-strong efficient. The second term $[f(\varepsilon,0) - f(0,0)]$ is contributed by the noise trading component when there is initial mispricing, and the last term $[f(\varepsilon,\beta) - f(\varepsilon,0)]$ is contributed by the feedback trading component when there is initial mispricing in the market. We will discuss these terms one by one in detail.

1. The profit contributed by both feedback trading and noise trading when the market is semi-strong efficient is given by

$$f(0,0) = \int_0^1 \sigma_t\sigma_v dt.$$

This term is an increasing function of both the level of noise trading, σ_t, and the uncertainty in the liquidation value, σ_v. The intuition is: In a semi-strong efficient market, the insider's profit is obtained from noise trading and the information advantage.

2. The insider's profit contributed only by the noise trading component when there is initial mispricing is given by

$$f(\varepsilon, 0) - f(0,0) = \int_0^1 \frac{\sigma_t \sigma_v \varepsilon^2 (1-t)^2}{[1+\varepsilon(1-t)]^2} dt.$$

This function increases for $\varepsilon > 0$, decreases for $\varepsilon < 0$, and reaches its minimum value zero at $\varepsilon = 0$.

- When the market is overpriced, with $\varepsilon \in (-1, 0)$, this term may go to very large because $[1+\varepsilon(1-t)]^2$ can approach zero. The intuition is that the closer the time is to one, the insider has higher certainty that the price will go back to the fundamental value and the trading against noise traders will generate higher profit.
- When the market is underpriced, with $\varepsilon > 0$, we have $1 + \varepsilon(1-t)^{\overline{\beta}(t)} > 0$ for all possible $\overline{\beta}(t)$ and ε. In this case,

$$\lim_{\varepsilon \to +\infty} \int_0^1 \frac{\sigma_t \sigma_v \varepsilon^2 (1-t)^2}{[1+\varepsilon(1-t)]^2} dt = \int_0^1 \sigma_t \sigma_v dt = f(0,0).$$

In summary, as ε goes from negative one to zero, this part decreases from the larger value (which is larger than $f(0, 0)$) to zero (its minimum value); then, as ε goes from zero to positive infinity, this part increases from zero to $f(0, 0)$.

The intuition is straightforward: the insider's profit from mispricing is zero when there is no mispricing. When there is mispricing, the insider can always benefit by introducing information into the market through trading. The profit comes from the insider's contribution to market efficiency.

3. The insider's profit contributed by the feedback trading component when there is initial mispricing can be given by

$$f(\varepsilon, \beta) - f(\varepsilon, 0) = \int_0^1 \sigma_t \sigma_v \left[\frac{\varepsilon^2 (1-t)^2 \phi^{\overline{\beta}(t)}}{\left(1+\varepsilon(1-t)\phi^{\overline{\beta}(t)}\right)^2} - \frac{\varepsilon^2 (1-t)^2}{[1+\varepsilon(1-t)]^2} \right] dt.$$

Because this is a zero-sum game, then the algorithm traders' profit is $-[f(\varepsilon, \beta) - f(\varepsilon, 0)]$.

1. For the feedback traders who correct the mispricing—that is, $\phi^{\overline{\beta}(t)} < 1$,[25]

$$f(\varepsilon, \beta) - f(\varepsilon, 0) < 0.$$

Feedback trading in the cases of (1) positive feedback trading when $\phi < 1$ or (2) negative feedback trading when $\phi > 1$ that corrects the mispricing lowers insider's profit. This implies that, in a market with initial mispricing, those who reduce the mispricing and make the price close to the efficient market price will make a positive profit, on average, as a reward. This result suggests that the level of feedback that reduces mispricing is positively associated with algorithm traders' profitability. In other words, when the algorithm traders can correctly predict the direction of price, then they can cut a share of the profit from the insider. From the perspective of the market, the presence of such feedback traders makes the market more efficient.

2. For the feedback traders who enlarge the mispricing —that is, $\phi^{\overline{\beta}(t)} > 1$,

$$f(\varepsilon, \beta) - f(\varepsilon, 0) > 0.$$

This suggests that algorithm traders in the cases of (1) $\phi > 1$ with positive feedback trading or (2) $\phi < 1$ with negative feedback trading strategies will lose money to the insider in the long run as a punishment which drives the price far way from the efficient market price.

In the situation of overpricing combined by (1) positive feedback trading when $\phi > 1$ or (2) negative feedback trading strategies when $\phi < 1$, the market depth parameter λ can go to infinity, indicating an extremely thin market; then, even small orders can trigger large price movements. In this case, the insider has additional opportunities to make a higher profit. The green line in Figure 6 demonstrates that this part may generate extremely high profit for the insider.

It is important to understand that the direction of the price process eventually moves toward the liquidation value. So, the average intensity of feedback that satisfies $\phi^{\overline{\beta}(t)} < 1$—that is, in the cases of (1) $\phi < 1$ with positive feedback trading or (2) $\phi > 1$ with negative feedback trading strategies—reflects the profitability of algorithm trading strategies.

As evidenced by Theorem 2, any mispricing will be eliminated in the end by the price discovery process. It follows that in a market with initial mispricing, algorithm trading that reduces (enlarges) mispricing, on average, will be profitable (losing money) as a reward (punishment) for improving (reducing) efficiency. There exists plenty of empirical evidence to support this result: in algorithm trading, the momentum factor suggests that stocks that have performed well in the past would continue to perform well (Jegadeesh and Titman 2001). The contribution of our work is that we explain: In the presence of mispricing, the market is becoming more efficient over time with insider trading, so, on average, people who adopt a feedback strategy reducing the mispricing would make a profit. In a sense, (1) "positive feedback trading" when $\phi < 1$ or (2) "negative feedback trading" when $\phi > 1$ studied in this paper is a measure of algorithm traders' capability of coming up with profitable strategies. In contrast, (1) "negative feedback trading" when $\phi < 1$ or (2) "positive feedback trading" when $\phi > 1$ indicate those who go against the fundamentals. If the asset is mispriced, algorithm trading in the case of (1) "negative feedback trading" when $\phi < 1$ or (2) "positive feedback trading" when $\phi > 1$ can hinder the process for information to be reflected in the price.

The feedback traders can change the informed trader's profit if and only if the market has already initially deviated from the semi-strong efficient market hypothesis. The informed trader always has (1) an informational advantage and, at the same time, (2) an extra mispricing advantage, regardless of the strategies that the algorithm trader takes (positive or negative). When the algorithm trader plays strategies that enlarge the mispricing—that is, (1) "negative feedback trading" when $\phi < 1$ or (2) "positive feedback trading" when $\phi > 1$—there is an additional opportunity for the informed trader to earn more profit from the feedback trading component. Meanwhile, although algorithm trading is at an informational disadvantage in the market, skillful algorithm traders by trading in the same direction with the informed trader can make a profit.

Our results also provide insights about learning in algorithm traders. Feedback traders who enlarge the mispricing—that is, in the case of (1) "negative feedback" when $\phi < 1$ or (2) "positive feedback" when $\phi > 1$—lose money in the long run in the process of the market becoming more efficient. If they are capable of learning, they will find that reducing mispricing feedback trading—that is, (1) "positive feedback" when $\phi < 1$ or (2) "negative feedback" when $\phi > 1$—is a more profitable strategy. When the market is composed of many such capable traders, there will be more feedback trading in the case of (1) "positive feedback" when $\phi < 1$ or (2) "negative feedback" when $\phi > 1$. Then, based on our results, we can expect the market to be more efficient, a desirable outcome for both the regulators and the market participants.

6. Conclusion

We present a model of algorithmic trading that incorporates initial mispricing, thereby relaxing the efficient market hypothesis and treating market efficiency as a continuous variable. We examine the market's response to such deviations and the effects of algorithmic trading on the market and its participants. The decision maker, an informed trader, maximizes profit in the face of potential asset price deviations by gradually releasing the information into the market through trading. This model allows us to investigate the conditions for market price convergence or divergence.

Our study yields several analytical results:

1. Initial mispricing is a necessary condition for the impact of algorithmic trading on price (Theorem 2).
2. The clearance price always converges to the liquidation value at market clearance, but the price trajectory may not follow the path of a semi-strong efficient market (see Figure 5).
3. We identify conditions leading to bubbles and crashes, showing that overpricing, combined with feedback trading that enlarges mispricing, has a more detrimental effect on market stability than underpricing (see Figures 2, 5, and 6 and Figures A.1 and A.2 in the online appendix).
4. We derive the profits of different market participants, showing that the informed trader earns more than in a semi-strong efficient market due to mispricing (see Figure 6). Noise traders lose more due to mispricing, whereas feedback trading that reduces mispricing makes a positive profit, as it indicates the traders' capability in generating alpha by following the informed trader.
5. Depending on whether the asset is underpriced or overpriced, different algorithm trading strategies may have different impacts on price process (Figure 5), informed trading profits (Figure 6), trading volume (Online Appendix A.1), and market depth (Online Appendix A.2).

We conclude that increased use of past-price-related algorithmic trading strategies based on big data and machine learning may increase market volatility, particularly in overpriced markets with feedback trading strategies that enlarge mispricing. Feedback trading that can mitigate the effect of initial mispricing is profitable with market deviation, whereas feedback trading that enlarges mispricing incurs more losses.

These results offer specific managerial implications for regulators and market participants.

For *policy makers*, it is important to understand that not all algorithm traders are bad for the market. Our results show that algorithm trading is a double-edged sword. It may reduce or increase market efficiency depending on (1) the direction of mispricing (i.e., overpricing or underpricing) and (2) whether the trading is in the right direction (i.e., leading to more or less price deviation). As a reward, algorithm trading that reduces pricing bias will be profitable, whereas algorithm trading that enlarges pricing bias will lose money as a punishment. The effect of feedback trading that enlarges mispricing depends on whether the market is underpriced or overpriced. Significant price volatility can *only* occur in an overpriced market. When overpricing happens, it is important for the regulators to provide liquidity to the market to avoid significant price fluctuations.

For the insider, any mispricing is associated with higher profit. Feedback trading that reduces mispricing decreases the profit, and feedback trading that enlarges the mispricing increases the profit. The aggregate effect depends on the aggregate feedback intensity. When overpricing and aggregate feedback that enlarge mispricing result in an illiquid market, the insider can leverage their informational advantage to significantly increase profits, albeit at the risk of causing market bubbles and crashes.

For feedback traders, the results suggest that even if feedback trading that enlarges mispricing may earn some profit in the short run, it loses money in the long run. Given the higher risk of trading in an overpriced market, feedback traders who enlarge mispricing and move price far away from the efficient level should be particularly careful in such a market with mispricing. Traders with self-learning capabilities should improve their strategies and aim to imitate the orders submitted by the insiders.

Finally, although the specific findings related to algorithmic trading are intriguing in their own right, we posit that the methodological contributions of this study—namely, the formulation of mispricing—and the exploration of algorithmic trading in an imperfectly efficient market could open new avenues in modeling how technology is transforming the financial market.

Acknowledgments

The authors thank Erik Brynjolfsson, Chris Dellarocas, Shane Greenstein, Jie Song, Tunay Tunca, Jiang Wang, Tan Wang, Jianming Xia, Feng Zhu, and the review panel for helpful discussions and valuable comments. Both authors contributed equally.

Endnotes

[1] https://fortune.com/2020/03/10/high-frequency-algorithmic-trading-stock-market-crash/.

[2] https://www.marketwatch.com/story/algorithms-sped-up-selling-leading-to-the-fastest-bear-market-in-stock-market-history-2020-03-26.

[3] The model in this paper also suggests that once mispricing is assumed away, there cannot exist bubbles and crashes. In other words, we show that mispricing is a necessary condition of extreme market volatility.

[4] https://www.washingtonpost.com/news/wonk/wp/2018/02/07/the-robots-v-robots-trading-that-has-hijacked-the-stock-market/.

[5] https://www.bloomberg.com/news/articles/2012-04-19/algorithmic-trading-may-spur-volatility-mispricing-turner-says.

[6] https://www.iosco.org/library/pubdocs/pdf/IOSCOPD354.pdf. IOSCO, established in 1983, unites global securities regulators and is acknowledged as the universal benchmark provider for the securities industry. The organization is devoted to creating, applying, and advocating for compliance with global standards in securities regulation. IOSCO collaborates closely with the G20 and the Financial Stability Board (FSB) to advance the international regulatory reform agenda. The organization's membership supervises over 95% of the world's securities markets across more than 130 jurisdictions.

[7] https://www.cnbc.com/2020/03/26/sec-pauses-zoom-technologies-as-traders-confuse-it-with-zoom-video.html.

[8] As will be clear, we do not assume the direction of the feedback, and in the framework, the feedback trading strategies are time-variant. Because there is no predictable trend, other participants cannot trade against it. There can be many algorithm traders with different trading strategies, based on different information in the market. As an outsider, it is difficult to profit from so many different strategies. We are able to focus on a representative agent because the orders can be aggregated together.

[9] Algorithm trading can also be triggered by events. But because events are exogenous and cannot be predicted, we exclude this type of algorithm trading from this research. In our study, we would like to focus on how algorithm trading strategies feed on themselves (through feedback trading) and potentially create market instability.

[10] There may be multiple algorithmic and nonalgorithmic traders, but the orders can be aggregated together. That is, if there are multiple traders, with every trader adopting a different feedback strategy (can be positive, negative, or not reacting to price at all), then, in aggregate, the representative agent still has a feedback strategy. See endnote 13 for a more detailed explanation with mathematics.

[11] Although there are many different kinds of algorithms in quantitative trading, most of such trading models depend on observations of time, volume, and price. Momentum strategies and most factors used in Fama-French type of factor models (Fama and French 1992) are all based on various ways of examining past prices.

[12] The zero mean assumption can be easily extended to a case with a positive mean to avoid negative asset values.

[13] The case of traders submitting limit orders can be shown to yield similar results.

[14] If there is no feedback trading, then the algorithm trader degenerates to the noise traders in Kyle (1985).

[15] We can examine the orders from the uninformed traders in an aggregate manner because these orders are additive. Suppose there are m algorithm traders and n noise traders. Then, the orders from these uninformed traders at time t can be aggregated as $dX_U(t) = \sum_{i=1}^m \beta_t^i \cdot dP_{t-} + \sum_{j=1}^n \sigma_{jt} \cdot dW_t^j$, where $(W_t^1, \ldots, W_t^n)$ is a n-dimensional standard Brownian motion, which can be simplified to Equation (3) after defining $\beta_t \equiv \sum_{i=1}^m \beta_t^i$ and $\sigma_t dW_t \equiv \sum_{j=1}^n \sigma_{jt} dW_t^j$.

[16] Market depth is formally defined as $1/\lambda_t$.

[17] Mathematically, $\{\mathcal{F}_t\}_{0\le t\le 1}$ is the natural filtration generated by the aggregate order process $\{X_I(t) + X_U(t)\}_{t\in(0,1)}$, with $\mathcal{F}_0 = \{\Omega, \phi\}$.

[18] The intensity of the feedback trading is determined by the algorithm traders according to past prices of the risky asset.

[19] Although the function $\theta(\cdot,\cdot)$ can be constructed as a multiplicative effect, it can be easily shown that the results remain the same if it is introduced as an additive effect.

[20] Note that β_t is a function of t. $\overline{\beta}$ is not a constant; it is the average feedback intensity of β_t over the whole duration $[0,1]$.

[21] For the case of $\phi > 1$, we can follow a similar discussion.

[22] Instantaneous feedback intensity β_t can change over time. We only need to know the average feedback intensity during the time interval from zero to t to derive the following results.

[23] For simplicity, we only plot the $\phi < 1$ case.

[24] That is, the cumulative feedback trading intensity $\tilde{\beta}(t)$ equals zero for all t.

[25] Note that here, β represents the function $\overline{\beta}(t)$, which measures the average feedback trading intensity for the time duration from zero to t.

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